Short Lecture Course

Modeling and Analysis of Dynamic Mechanical Systems

with a special focus on

Rotordynamics and

Active Magnetic Bearing (AMB) Systems

R. Larsonneur
Mecos Traxler AG, 8404 Winterthur, Switzerland
rene.larsonneur@mecos.com
www.mecos.com

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Preface

Active magnetic bearings (AMB) have become increasingly important during the past two decades and are nowadays installed in a large number of industrial applications in turbo machinery, vacuum technology and energy production. Due to the complete lack of any mechanical contact AMBs offer special properties that cannot be achieved by other bearings technologies. The most important among those special features are the lubricant-free and, therefore, contamination-free operation, the ability to be operated at very high speeds and the possibility for active vibration control (damping of mechanical oscillations, compensation of unbalance forces, etc.). Moreover, as they are inherently equipped with sensors and active control elements, AMBs offer a built-in monitoring and supervision capability, without any need for additional instrumentation. This opens new possibilities for early fault diagnosis and preventive maintenance, hence rendering such a system a “smart machine”.

Active magnetic bearings are a typical mechatronic product. This means that, in order to understand, develop and use this technology, the behavior of each single system component (rotor, bearing, sensor, controller, etc.), and the various interactions between them must be investigated first.

The aim of this short lecture course is to investigate the behavior of the most important component of any AMB system more in detail: the levitated body. Here, the course restricts itself to rotating bodies (rotors). Starting from a very simple dynamic mechanical system the modeling and analysis tools are developed and then extended to the special properties of a rotating mechanical system. The results are compared with actual measurements obtained from an experimental AMB test rig.

1. A Simple 1 DOF System

1.1. Introduction

In order to analyze a complex dynamic mechanical system it is essential to first understand the properties of the simplest possible dynamic mechanical system: a system with only one degree of freedom (DOF). Based on this it can be shown that the modeling and analysis concepts developed for such a simple system can rather straightforwardly be adapted to a system with more than one DOF (the number of DOF is the minimum number of coordinates necessary to describe the entire system state).

In practice, one of the most important aspects of a dynamic mechanical system is its ability to oscillate. Most often, the resulting vibrations are unwanted effects and have to be suppressed. Hence, emphasis in this course is put on vibration analysis.

1.2. The Undamped Free 1 DOF Mechanical Oscillator

1.2.1. Equation of Motion

The simplest possible mechanical structure with the ability to oscillate is a single rigid mass attached to a spring (spring-mass system). It is known that, for any kind of oscillatory system, there must be different energy reservoirs between which energy can be transferred in both directions. In the case of the simple spring-mass system there are two such reservoirs: Kinetic energy is stored in the mass when it moves, potential energy is stored in the spring when it is compressed or expanded. During oscillation energy is converted and transferred back and forth between these two reservoirs, in the simplest (theoretical) case without any loss.

In Figure 1.1 a simple spring-mass system is displayed. The mass m is assumed to carry out vertical translational movements only along the direction x, the spring is assumed to have no mass but a stiffness k. It is furthermore assumed that there is no other external force than the weight of the mass.

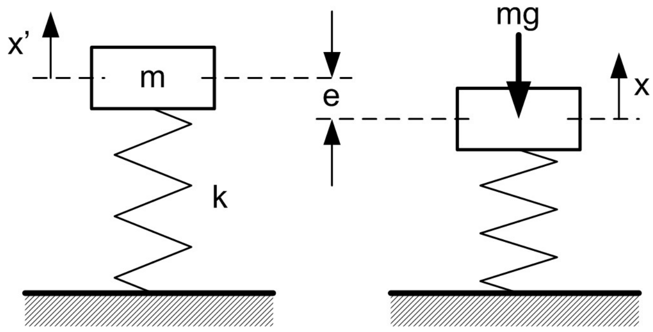

Figure 1.1: Undamped free 1 DOF spring-mass system

The equations of motion for this simple system can be derived by energy methods or by Newton's resp. Euler's laws, which is the simplest approach here:

$$m\,\ddot{x}' = -k\,x' - m\,g \tag{1.1}$$

In equation (1.1) it is adopted that that the x' coordinate has its origin where the spring is unloaded. This has the effect that the constant weight term mg still exists within the inhomogeneous differential equation (DE). It turns out that it is more useful to define another inertial coordinate x, having its origin at the point where the spring is loaded (compressed) by the weight. The static equilibrium condition for this case results to

$$k\,e = m\,g \tag{1.2}$$

and DE (1.1) together with (1.2) becomes simpler:

$$m\,\ddot{x} = -k\left(x - e\right) - m\,g \qquad \leftrightarrow \qquad m\,\ddot{x} = -k\,x \tag{1.3}$$

The result from (1.3) can be brought into the well-known simplest form of a homogeneous 2nd order DE for an oscillator:

$$\boxed{m\,\ddot{x} + k\,x = 0} \tag{1.4}$$

1.2.2. Vibration Analysis

Apart from the trivial solution, which is x=0, we are first of all interested in a non-vanishing solution x(t) that satisfies DE (1.4). From mathematics we know that the following guess function always yields a solution for this class of DE (homogeneous, linear, constant coefficients):

$$x(t) = e^{\lambda t} \tag{1.5}$$

By introducing this guess into DE (1.4) we obtain:

$$m\lambda^2 e^{\lambda t} + k e^{\lambda t} = 0 \qquad \leftrightarrow \qquad \left(m\lambda^2 + k\right) e^{\lambda t} = 0 \tag{1.6}$$

Equation (1.6) is called the "characteristic equation" and the expression in brackets is the "characteristic polynomial" p(λ) of the system. λ is called the system's "eigenvalue".

Since, for any value of λ, the term $e^{\lambda t}$ never yields 0 the guess (1.5) results in a condition that must be satisfied by the still unknown eigenvalue λ

$$\lambda_{1,2} = \pm\sqrt{-\frac{k}{m}} = \pm j\sqrt{\frac{k}{m}} \tag{1.7}$$

and, hence, the general solution of DE (1.4) becomes the superposition of the two solutions for the guess function found:

$$x(t) = C_1 e^{j\sqrt{\frac{k}{m}}t} + C_2 e^{-j\sqrt{\frac{k}{m}}t} \tag{1.8}$$

However, we must claim that the solution x(t) is non-imaginary. This can easily be achieved by the following mathematical equivalences:

$$\cos(\varphi) = \frac{e^{j\varphi} + e^{-j\varphi}}{2}; \quad \sin(\varphi) = \frac{e^{j\varphi} - e^{-j\varphi}}{2j} \tag{1.9}$$

With (1.9) the general solution (1.8) becomes a purely real solution with real constants A and B and angular frequency ω_0:

$$x(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t);\ \omega_0 = \sqrt{\frac{k}{m}} \tag{1.10}$$

Equation (1.10) is the classic description of an undamped free oscillatory system with the angular frequency ω_0 (unit rad s^{-1}). As already mentioned, this angular frequency is also called the “eigenfrequency”, a term which is of greatest importance for any vibration system. The physical meaning of the eigenfrequency is very intuitive: it describes the period of the oscillation which is proprietary to the system itself, hence only determined by its parameters (“eigen” means “proprietary” in German language). In the case of the simple spring-mass system this eigenfrequency is only determined by the spring stiffness k and the mass m. The stiffer the spring or the smaller the mass, the higher the eigenfrequency will be – a result which corresponds nicely to everybody’s experience.

The still undefined constants A and B are determined by the initial condition of the oscillation, i.e. with which amplitude and which initial velocity of the mass m the oscillation is started. The simplest case is given if the motion is started with a given ampitude x_0 and vanishing initial velocity. In this case the final solution of DE (1.4) becomes:

$$x(t) = x_0\cos(\omega_0 t);\ \omega_0 = \sqrt{\frac{k}{m}};\quad x(0) = x_0;\quad \dot{x}(0) = 0; \tag{1.11}$$

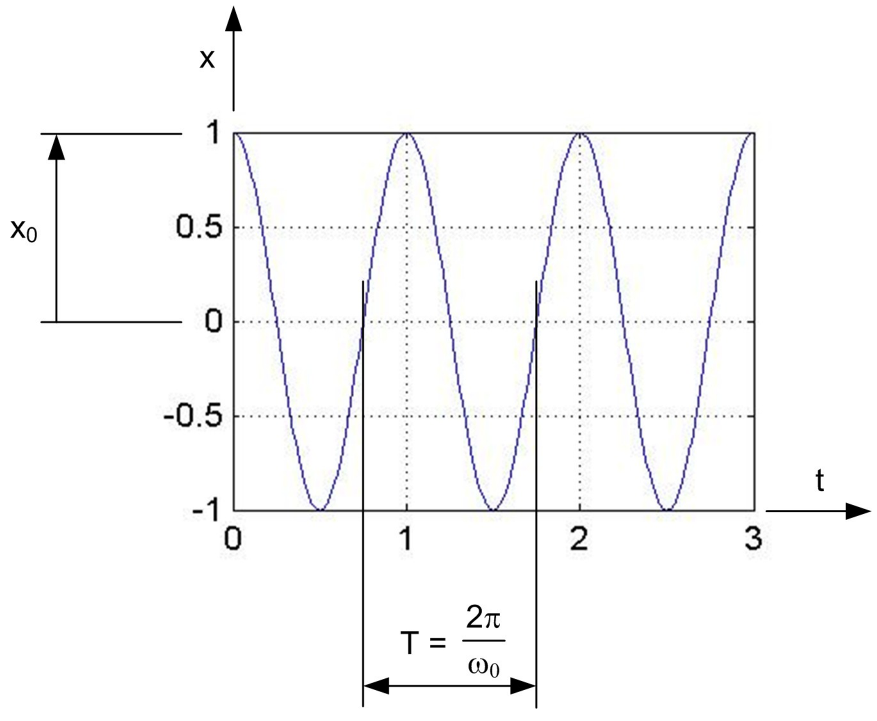

Figure 1.2: Oscillation of the undamped 1 DOF spring-mass system

As can be seen from equation (1.11) resp. from figure 1.2, the undamped or also called “harmonic” oscillation will never stop once excited, hence, energy will be transferred back and forth between the mass and the spring without any loss. Due to the fact that the total amount of energy is maintained constant within the system, such as system is also called "conservative”. This can very easily be verified by setting up the total energy E as the sum of kinetic and potential energy followed by taking the time derivative of this expression:

$$E = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 \tag{1.12}$$

$$\dot{E} = \frac{d}{dt}\left(\frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2\right) = m\dot{x}\ddot{x} + kx\dot{x} = (m\ddot{x} + kx)\dot{x} = 0 \tag{1.13}$$

As we can see in expression (1.13) the time derivative of the total energy E is zero, hence, E must be constant, which proves the assumption of energy conservation.

Energy conservation is a strongly idealized case – in reality there is always some amount of energy dissipation associated with any mechanical system vibration. Therefore, a simple dissipation element is added to the system in the next section.

1.3. The Damped Free 1 DOF Mechanical Oscillator

1.3.1. Equation of Motion

A very suitable mechanical model for dealing with energy losses is the viscous damper, provided, that displacements and velocities are small (linear system). The viscous damper produces a force which is proportional to the velocity of the elements within the damper and opposite to the instantaneous direction of motion. In figure 1.3 such a viscous damper with damping constant d is added to the simple spring-mass system.

The equation of motion can be derived in exactly the same way as shown in paragraph 1.2.1:

$$m\,\ddot{x} = -k\,x - d\,\dot{x} \tag{1.14}$$

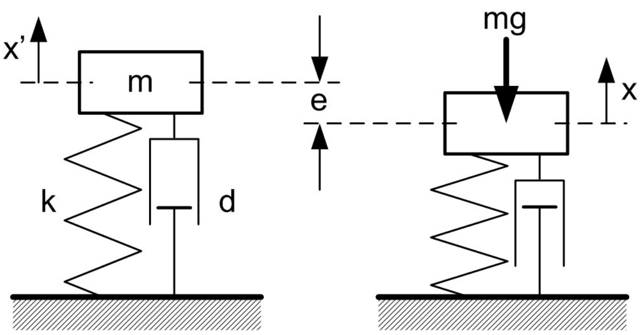

Figure 1.3: Free 1 DOF spring-mass system with viscous damper

As x is still the coordinate with origin in the position of the spring loaded by the weight the term mg vanishes from equation (1.14). As before the resulting DE is again of 2nd order, homogeneous and has constant coefficients:

$$\boxed{m\ddot{x} + d\dot{x} + kx = 0} \tag{1.15}$$

1.3.2. Vibration Analysis

The derivation of the solution of DE (1.15) is similar to the procedure for the undamped system. Here, the characteristic equation for the eigenvalue λ yields the characteristic polynomial:

$$m\lambda^2 + d\,\lambda + k = 0 \tag{1.16}$$

For the following analysis it is suitable to use expressions already developed for the undamped system. By doing this equation (1.16) becomes:

$$\lambda^2 + 2\sigma\,\lambda + \omega_0^2 = 0; \qquad \omega_0^2 = \frac{k}{m}; \;\; \sigma = \frac{d}{2m} \tag{1.17}$$

As we can see, the solution of the characteristic equation (1.17), i.e. the eigenvalue λ that determines the eigenfrequency ω of the system, now also depends on the damping factor d resp. on the normalized damping coefficient σ:

$$\lambda_{1,2} = -\sigma \pm j\sqrt{\omega_0^2 - \sigma^2} \tag{1.18}$$

From equation (1.18) we can see that, with increasing damping, the eigenvalue λ is no more purely imaginary as given by (1.7) but becomes generally complex with a negative real part equivalent to the damping coefficient σ. This behavior, i.e. the smooth transition from an undamped to an increasingly damped system, can also be described by a root plot of λ within the complex plane, as shown in figure 1.4.

In figure 1.4 it can easily be seen that, with increasing damping, a negative real part of the solution λ develops, while the imaginary part becomes smaller. For large damping coefficients, i.e. for $\sigma >= \omega_0$, the imaginary part is identically 0 and two real solutions for λ exist, as also viewable from (1.18). In this case the system reaches “critical” resp. “overcritical” damping. In this short course, only “undercritical” damping is considered, This means, that such systems maintain their oscillation capability, whereas critically or overcritically damped systems cannot oscillate any more.

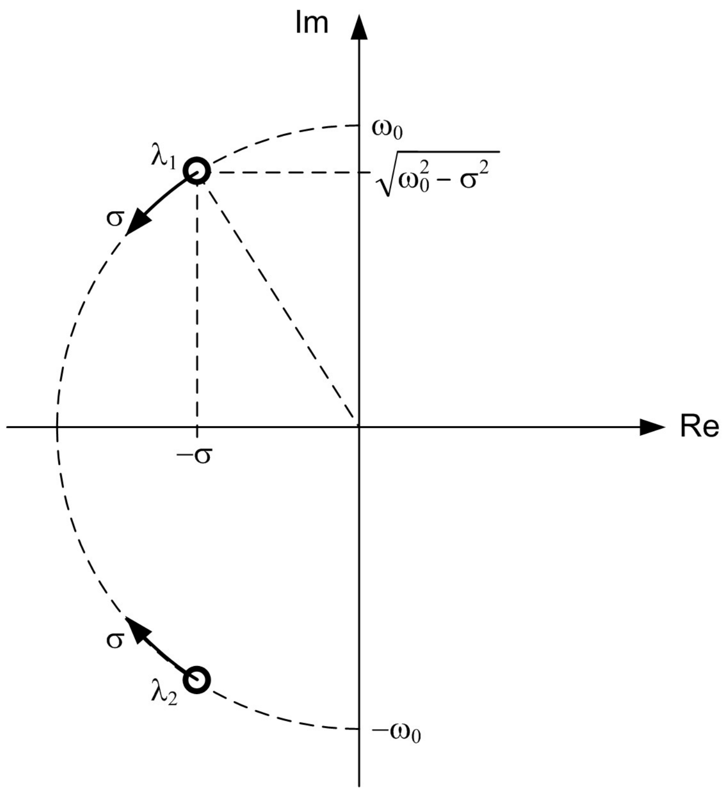

Figure 1.4: Root locus of the eigenvalue λ of the damped 1 DOF system as a function of the damping coefficient σ

By inserting the solution (1.18) of the characteristic equation into the guess function (1.5) and by developing a purely real solution for the displacement x(t), as done for equation (1.10) in the undamped case, we obtain the general solution for the time behavior of the damped spring-mass system:

$$x(t) = e^{-\sigma t}\left(A\cos(\omega t) + B\sin(\omega t)\right);\ \ \omega = \sqrt{\omega_0^2 - \sigma^2};\ \ \sigma < \omega_0 \tag{1.19}$$

Equation (1.19) describes a motion with an exponentially decaying amplitude, hence, this oscillation can no more be called harmonic or even periodic, as it was the case for the undamped system. Nevertheless, it still makes sense to consider the system an oscillatory system, especially for small damping coefficients. This becomes even clearer when the time between two consecutive zero-crossing points is examined (initial conditions as given in section 1.2.2). This is shown in figure 1.5:

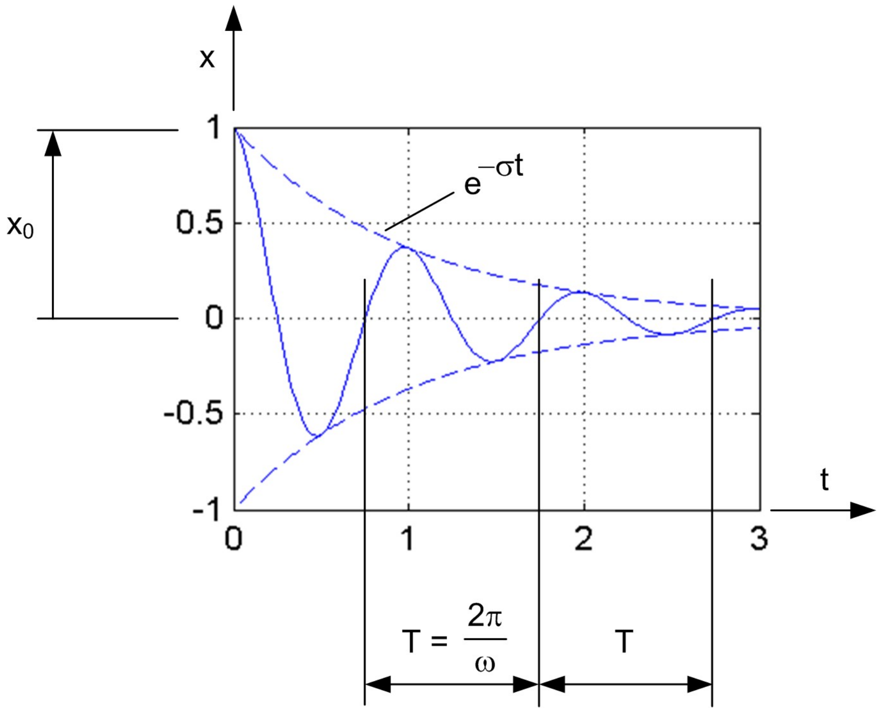

Figure 1.5: Damped oscillation of the 1 DOF spring-mass system

As can be seen from figure 1.5 the time T between the zero-crossing points is constant, even if the oscillation amplitude becomes smaller. However, it is important to mention that this does not hold for the time between two relative amplitude maxima. The time T can, therefore, be called a "pseudo period" of the damped system, and ω can correspondingly be called the "pseudo angular frequency". This relationship is given in equation (1.20):

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \sigma^2}} \tag{1.20}$$

The pseudo angular frequency ω, which is, by definition, the so-called "eigenfrequency" of the damped system, is smaller than the frequency ω_0 in the undamped case, hence, it can be concluded that any additional damping reduces the eigenfrequency of an originally undamped system. This effect is also depicted in the root plot in figure 1.4.

We will see further below that the negative real part of the eigenvalue λ determines the stability of the system. In the undamped case the system is only "limit stable", also recognizable by the non-decaying motion amplitude (see figure 1.2). In the damped case, however, the motion amplitude asymptotically reaches zero with time, and the system can be considered "asymptotically stable".

1.4. Forced Vibration of a Damped 1 DOF Mechanical Oscillator

Up to now we have only considered the so-called "free" system, i.e. external forces have not been included in the model description (apart from the weight). However, the influence of external forces is present and of great importance in any practical mechanical system (e.g. unbalance forces in a rotating machine). Therefore, it clearly makes sense to investigate the influence of an external force F(t), as introduced in figure 1.6, on the simple 1 DOF system developed so far.

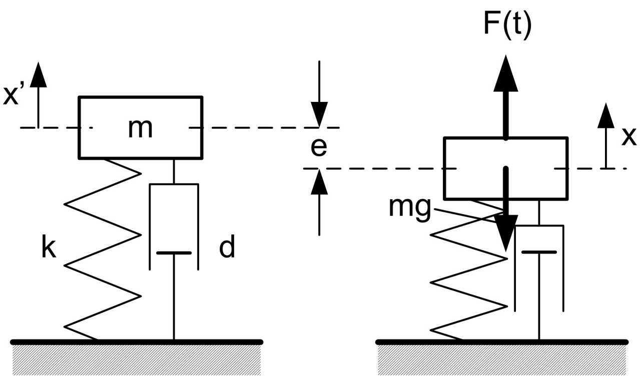

Figure 1.6: 1 DOF damped spring-mass system with external disturbance force

1.4.1. Equation of Motion

The equation of motion, as developed above, can be simply expanded by the term for the external force:

$$m\,\ddot{x} = -k\,x - d\,\dot{x} + F(t) \tag{1.21}$$

The previously homogeneous differential equation (DE) now becomes inhomogeneous:

$$m\ddot{x} + d\dot{x} + k\,x = F(t) \tag{1.22}$$

From mathematical theory of linear DE we know that the most general solution of (1.22) is always a superposition of the solution (1.19) of the homogeneous DE and a particulate solution of the inhomogeneous DE (1.22). Since, in most cases, we are not so much interested in the transient system behavior but rather in the system's response to the external force, we can neglect the homogeneous part of the solution, provided that we deal with an asymptotically stable system for which the transient part of the response will always decay to zero after some time.

The particulate solution to the inhomogeneous DE (1.22), however, strongly depends on the time dependency of the external force. If we don't know this dependency the solution (integration) of the DE is impossible. In practice, external forces are very often periodic, especially in rotating machinery. In this case F(t) can be considered a superposition of harmonic functions with different frequencies, but all frequencies being integer multiples of the periodic force's fundamental frequency (so-called "Fourier" decomposition). With the property that the dynamic system itself is linear it's response to such a periodic external force is the superposition of its responses to the individual harmonic components of the external force. Thus, it becomes clear that the analysis of the purely harmonic system response is fundamentally important and serves for solving the more general case of any periodic external excitation.

Following this finding, we can replace the term for the force F(t) in (1.22) by a purely harmonic term with frequency Ω:

$$m\ddot{x} + d\dot{x} + k\,x = F\cos(\Omega t) \tag{1.23}$$

For the further analysis it is suitable to divide equation (1.23) by the mass m in order to achieve a description using the above introduced characteristic parameters σ and ω_0. We will also make use of the fact that the systems investigated here are assumed to be undercritically damped ($\sigma < \omega_0$).

$$\boxed{\ddot{x} + 2\sigma\dot{x} + \omega_0^2\,x = f\cos(\Omega t); \qquad \omega_0^2 = \frac{k}{m};\; \sigma = \frac{d}{2m}} \tag{1.24}$$

1.4.2. Vibration Analysis

The particulate solution $x_P(t)$ of (1.24) can be either obtained by a complex or by a purely real mathematical analysis. It turns out, that complex analysis is more elegant and efficient, whereas the purely real approach is more straightforward and avoids the introduction of virtually complex external excitation forces.

A simple and always working guess for the particulate solution of DE (1.24) is to set up a general response similar to the excitation, i.e. by assuming that the response is also harmonic, with identical frequency Ω but with an arbitrary phase shift φ between excitation and system response and with an amplification factor a. Hence, the mass m no longer oscillates with its eigenfrequency ω but with the same frequency Ω as the excitation force. This behavior is called the “forced vibration” of the damped spring-mass system.

$$x_P(t) = a\, x_0 \cos(\Omega t - \varphi) \tag{1.25}$$

In expression (1.25) x_0 is the static displacement caused by a force with amplitude f, hence:

$$x_0 = \frac{f}{\omega_0^2} \tag{1.26}$$

By inserting expressions (1.25) and (1.26) into DE (1.24) one obtains the following relationships for the amplification a and the phase shift φ:

$$\boxed{\tan(\varphi) = \frac{2\sigma\Omega}{\omega_0^2 - \Omega^2}} \tag{1.27}$$

$$\boxed{a = \frac{\omega_0^2}{\sqrt{\left(\omega_0^2 - \Omega^2\right)^2 + (2\sigma\Omega)^2}}} \tag{1.28}$$

For the graphical plot of above expressions it is very useful to introduce the dimensionless frequency κ and the dimensionless damping coefficient ε:

$$\kappa = \frac{\Omega}{\omega_0}; \quad \varepsilon = \frac{2\sigma}{\omega_0} \tag{1.29}$$

Expressions (1.27) and (1.28) then become with (1.29):

$$\tan(\varphi) = \frac{\varepsilon\kappa}{1-\kappa^2} \tag{1.30}$$

$$a = \frac{1}{\sqrt{\left(1-\kappa^2\right)^2 + (\varepsilon\kappa)^2}} \tag{1.31}$$

It is most interesting to investigate the phase shift φ between the external force and the displacement $x_p(t)$ as a function of the dimensionless frequency κ, i.e. for different excitation frequencies. These relationships are plotted in figure 1.7 for different dimensionless damping coefficients ε.

In figure 1.8 the time response $x_P(t)$ and the excitation force f(t) are plotted over time for different dimensionless frequencies κ.

The analysis of the behavior shown in figure 1.7 and 1.8 brings up the following results:

The phase shift φ between excitation and system response rises with the excitation frequency Ω. For low frequencies there is almost no phase shift (force and vibration are in phase), whereas at very high frequencies the phase shift is π, hence, the oscillation of the mass m is in counter phase to the excitation.

Most interesting is the system behavior for κ=1, i.e. if the excitation frequency Ω is equal to the eigenfrequency ω_0 of the undamped system. This case is called “resonance”, one of the most important phenomena known for oscillatory systems of any kind (not necessarily only for a mechanical system).

It is important to mention that the definition of resonance is not made based on the maximum response amplitude (amplification), as most often wrongly assumed, but on the phase shift φ. By definition resonance occurs when the phase shift φ is π/2.

The most known and feared effect from resonance is the large amplification of the system response (see figure 1.7). For very small damping coefficients a huge system response can occur even if the excitation force is very small. This can lead to a system collapse due to an excessive vibration amplitude.

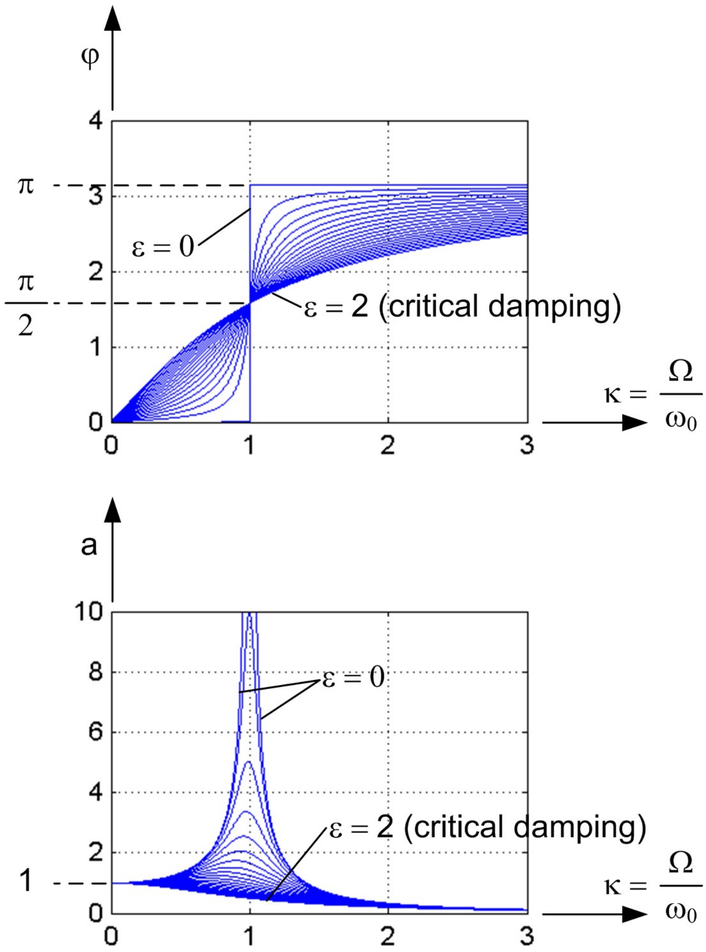

Figure 1.7: Phase shift φ and amplification a as a function of the dimensionless frequency κ for different damping coefficients ε

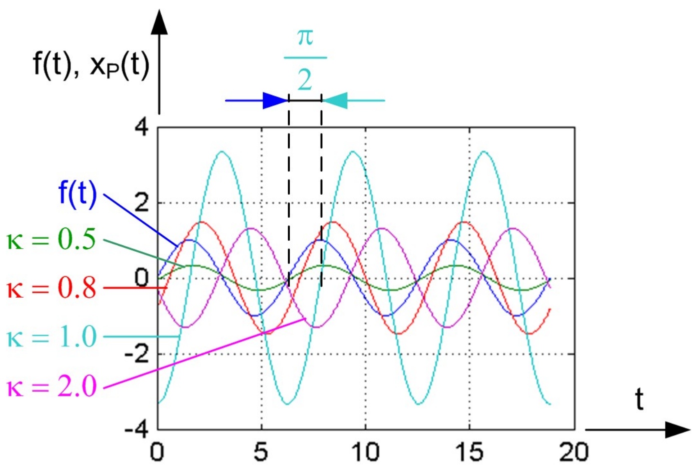

Figure 1.8: External force f(t) and oscillation $x_P(t)$ for κ = [0.5, 0.8, 1, 2] ($\varepsilon = 0.3$, ω_0 = 1 rads^{-1})

For reasonable damping coefficients ε the resonance phenomenon becomes weaker, i.e. the amplification factor a does not become infinite any more as in the case of ε=0. However, amplification still has a maximum near the resonance frequency. Note, that this maximum always occurs at a frequency slightly below the resonance frequency (κ<1), a fact which can also be seen in figure 1.7.

1.4.3. Generalization of the Frequency Response

In the previous section we have derived the phase shift φ and the amplification a of the system response of the simple 1 DOF damped oscillator:

$$\tan(\varphi) = \frac{2\sigma\Omega}{\omega_0^2 - \Omega^2}; \qquad a = \frac{\omega_0^2}{\sqrt{\left(\omega_0^2 - \Omega^2\right)^2 + (2\sigma\Omega)^2}} \qquad (1.27/1.28)$$

This finding can be generalized by a transformation of the time domain signals into the frequency domain by means of the so-called “Laplace” transform. Without any proof this is shortly outlined here.

Starting from the differential equation (1.24)

$$\ddot{x} + 2\sigma\,\dot{x} + \omega_0^2\,x = f\cos(\Omega t); \qquad \omega_0^2 = \frac{k}{m};\ \ \sigma = \frac{d}{2m} \qquad (1.24)$$

we can carry out the Laplace transform. This step transforms the DE into a polynomial equation in the complex frequency variable s:

$$s^2X(s) + 2\sigma\,sX(s) + \omega_0^2\,X(s) = F(s); \qquad \omega_0^2 = \frac{k}{m};\ \ \sigma = \frac{d}{2m} \qquad (1.32)$$

The transfer function G(s) is defined as the quotient X(s)/F(x), hence:

$$G(s) = \frac{X(s)}{F(s)} = \frac{1}{s^2 + 2\sigma s + \omega_0^2} \qquad (1.33)$$

The complex frequency response G(jΩ:) is the transfer function evaluated along the imaginary axis, hence for s = jΩ. This yields:

$$G(j\Omega) = \frac{1}{\omega_0^2 - \Omega^2 + j2\sigma\Omega} \tag{1.34}$$

By comparison of expression (1.34) with (1.27/1.28) one can directly see the very close relationship between the expression for the general frequency response G(jΩ) known from control theory and the above developed expressions for the phase shift φ and the amplification factor a:

$$\varphi = -\arg\{G(j\Omega)\}; \qquad a = |G(j\Omega)|\,\omega_0^2 \tag{1.35}$$

The "correction" factor ω_0^2 simply comes in by the fact that the quantity a was defined as an amplification of the static displacement x_0 (equation 1.26), whereas the frequency response links a force to a displacement.

To summarize:

Instead of deriving the phase shift and the amplification from the time domain solution – a rather cumbersome approach for more complex systems – one can simply derive the same quantities directly from the complex frequency response, a much more elegant, fast and general approach. As we will see in the next chapter this will also hold for dynamic mechanical systems with more than 1 degree of freedom.

2. Systems With More Than 1 DOF

2.1. Introduction

In reality dynamic mechanical systems always have more than only 1 DOF. In fact, if one takes a closer look, such systems even must be considered having an infinite number of DOFs, since they must be understood as a so-called “continuum”.

While there are in fact specialized analytic methods for dealing with a continuum, it turns out to be very impractical to consider an infinite number of DOFs. It is a much more suitable and practical approach to first get clarity about the question of how many DOFs must be considered to be able to describe the physical effects that one is interested in. Hence, for example, if one is only interested in looking at a so-called “rigid-body” behavior of a structure, it might be enough to consider a 6 DOF freedom system only, whereas it will be necessary to consider a higher number of DOF if one also wants to include flexible modes that a structure always has.

In the next sections various dynamic mechanical systems with a different number of DOF are analyzed with the aim of developing a more “generalized view” for systems with an arbitrary number of degrees of freedom.

2.2. Undamped Free System with 2 DOF

A 2 DOF system can be obtained by a straightforward expansion of the simple spring-mass system to two masses and two springs, as shown in figure 2.1.

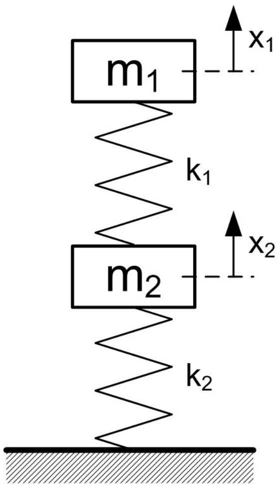

Figure 2.1: Undamped free 2 DOF spring-mass system

2.2.1. Equation of Motion

The equations of motion can be derived analogously to the 1 DOF case. It is again assumed that the inertial coordinates have their origin in the position where the springs are loaded by the weights.

For each of the two masses Newton's law can be formulated:

$$m_1\,\ddot{x}_1 = -k_1\left(x_1 - x_2\right) \tag{2.1}$$

$$m_2\,\ddot{x}_2 = +k_1\left(x_1 - x_2\right) - k_2 x_2 \tag{2.2}$$

The two expressions can be combined into a matrix formulation:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix}\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.3}$$

As for the 1 DOF system we obtain a linear differential equation of 2nd order with constant coefficients, however in this case as a matrix equation of dimension two.

The two displacements x_1 and x_2 can be combined in a displacement vector **x**. By doing this one also directly gets matrices **M** and **K**:

$$\mathbf{x} = \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix};\ \mathbf{M} = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix};\ \mathbf{K} = \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \tag{2.4}$$

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}} \tag{2.5}$$

Matrix **M** denotes the mass matrix of the system, whereas **K** is the stiffness matrix. Otherwise, DE (2.5) is almost identical to the corresponding expression (1.4) of the 1 DOF case with the only difference that instead of scalar terms matrices and vectors are involved.

2.2.2. Vibration Analysis

It can be shown that a multi-dimensional homogeneous DE as given in (2.5) can be solved in basically the same way as it was done for the 1 DOF system, i.e. by introducing a suitable guess function. However, since the solution must allow for different vibration amplitudes for each of the masses, a vector $\tilde{\mathbf{x}}$ containing those amplitudes as its components must be introduced as well:

$$\mathbf{x}(t) = \begin{Bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{Bmatrix} e^{\lambda t} = \tilde{\mathbf{x}} e^{\lambda t} \tag{2.6}$$

With (2.6) inserted into expression (2.5) one obtains the following matrix expression:

$$\left(\lambda^2 \mathbf{M}\tilde{\mathbf{x}} + \mathbf{K}\tilde{\mathbf{x}}\right) e^{\lambda t} = \mathbf{0} \tag{2.7}$$

Here again, since the exponential function never yields 0, expression (2.7) simplifies to:

$$\left(\lambda^2 \mathbf{M} + \mathbf{K}\right)\tilde{\mathbf{x}} = \mathbf{0} \tag{2.8}$$

It is clear that the zero vector $\tilde{\mathbf{x}} = 0$ satisfies (2.8) which, however, is not what we are interested in. Instead of this so-called trivial solution we are looking for a non-zero solution for $\tilde{\mathbf{x}}$. From linear algebra we know that this is only possible if matrix ($\lambda^2\mathbf{M}+\mathbf{K}$) is not regular, i.e. cannot be inverted. The mathematical formulation of this postulation is that the matrix's determinant must vanish:

$$\det\left(\lambda^2 \mathbf{M} + \mathbf{K}\right) = \mathbf{0} \tag{2.9}$$

As for the 1 DOF case equation (2.9) is an equation for the determination of those values of λ that allow for a solution of (2.8). Hence, expression (2.9) constitutes the "eigenvalue problem" for the 2 DOF case. The characteristic equation for λ resulting from writing down the determinant in (2.9) results to:

$$m_1 m_2 \lambda^4 + \left(m_1\left(k_1 + k_2\right) + m_2 k_1\right)\lambda^2 + k_1 k_2 = 0 \tag{2.10}$$

Equation (2.10) is a so-called “biquadratic” expression in λ, hence, the four solutions can be found analytically. However, instead of deriving those here, more emphasis is put on the generalization of (2.10) resp. of the eigenvalue problem (2.8) resp. (2.9). This approach is beneficial since, as we will see further below in a more general case, the eigenvalue problem results in most cases in an polynomial expression in λ which cannot be solved analytically.

A generalization of (2.8) can be found by multiplying the entire expression with the inverse of the mass matrix **M** and by rearranging the entire expression. One can show that the inverse of **M** always exists for mechanical vibration systems (all masses are positive, matrix **M** is symmetric and positive definite). We obtain:

$$\boxed{\left(-\mathbf{M}^{-1}\mathbf{K}\right)\tilde{\mathbf{x}} = \lambda^2\tilde{\mathbf{x}}} \tag{2.11}$$

Expression (2.11) denotes the special eigenvalue problem for a conservative mechanical system. Here, the values of λ are the eigenvalues of the matrix $-\mathbf{M}^{-1}\mathbf{K}$. For every eigenvalue λ a so-called eigenvector $\tilde{\mathbf{x}}$ exists so that (2.11) is satisfied. The number of existing solutions for λ and associated eigenvectors $\tilde{\mathbf{x}}$ is always twice the dimension n of the matrix $-\mathbf{M}^{-1}\mathbf{K}$, hence, for the given 2 DOF system, this quantity is 4. For the conservative case (2.11), all eigenvalues λ exist as complex conjugate pairs with vanishing real part. The relationship between eigenvalue and eigenfrequency is given by (2.12):

$$\boxed{\lambda_i = j\omega_i; \quad i = 1...2n} \tag{2.12}$$

The eigenvalue problem of the 1 DOF case can be considered nothing else than a special case of (2.11). This can easily be verified by comparison with the corresponding expressions (see chapter 1). Also the number of solutions, 2 in the 1 DOF case, corresponds to the above findings.

The abstract geometric relevance of (2.11) is the following: For every matrix, here for the matrix $-\mathbf{M}^{-1}\mathbf{K}$, there exist unique vectors, the eigenvectors $\tilde{\mathbf{x}}$, so that the vector obtained by mapping with the matrix, hence the product of the matrix and the vector, point in the same direction as the original vector and is scaled by the eigenvalue. This is not self-evident since, for an arbitrary matrix and an arbitrary vector, the product of the two is a vector that normally points in a different direction, hence corresponds to a scaling plus a rotation of the original vector. For the eigenvector, however, only a scaling takes place when multiplied with its corresponding matrix.

Apart from its geometric meaning the eigenvalue problem (2.11) is of greatest importance in every vibration system (not only mechanical), since it constitutes the conditional equation system for the eigenfrequencies and the eigenvectors. As we can see the eigenfrequencies and eigenvectors are only determined by the properties of the system, here by the mass matrix **M** and stiffness matrix **K**.

In the 1 DOF case the eigenvector didn't show up directly, since only one coordinate was considered. In the 2 DOF case, as in the general case, the eigenvector associated with every eigenvalue becomes very important. It contains the physical size of each component relative to all other components, hence, the form of the vibration. Therefore, the eigenvector is also called "mode shape" or "eigenmode".

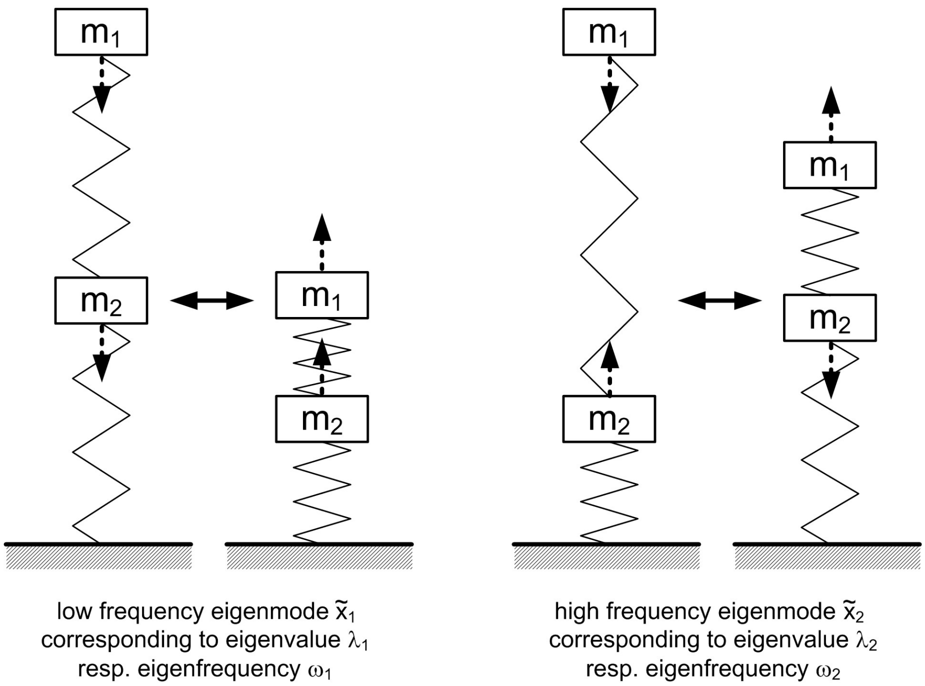

Figure 2.2: Qualitative sketch of the eigenmodes of the 2 DOF spring-mass system

In figure 2.2 the eigenmodes for the two resulting eigenfrequencies of the system are qualitatively shown. When oscillating in the low frequency eigenmode the two masses basically move up and down in phase, whereas in the high frequency eigenmode the two masses move in counter phase. Common to any eigenmode, however, is the fact that all masses involved oscillate with the same frequency, the eigenfrequency. The general oscillation is given by a superposition of the two eigenmodes (see also equation 2.25).

2.3. Damped Free System with 2 DOF

As done for the simple 1 DOF spring mass system viscous dampers can be added to the 2 DOF system in order to obtain a more practical model, hence the system will be no more conservative. If the dampers are mounted in parallel to the springs the following matrix DE results:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} d_1 & -d_1 \\ -d_1 & d_1 + d_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.13}$$

Analogously to the damped 1 DOF system we obtain the so-called damping matrix **D** which, in this case, has the same structure as the stiffness matrix **K** (not so in general). Equation (2.13) can also be written as

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0}} \tag{2.14}$$

The search for the solution of (2.14) leads us again to the eigenvalue problem, i.e. the equation system for the definition of eigenvalues (eigenfrequencies) and corresponding eigenvectors (eigenmodes):

$$\left(\lambda^2 \mathbf{M} + \lambda \mathbf{D} + \mathbf{K}\right)\tilde{\mathbf{x}} = \mathbf{0} \tag{2.15}$$

The characteristic polynomial resulting from the general eigenvalue problem (2.15) by formulating the zero determinant condition will not be biquadratic any more, hence, it will be a general polynomial of order 4 in λ. An analytical solution to this problem is difficult to find and, consequently, numerical methods will have to be applied. For doing this (2.15) must be brought into the special form of an eigenvalue problem, as done in (2.11). However, by left multiplication with the inverse of the mass matrix **M**, one does not obtain such a desired form. We will see further below (section 2.5.2) how this eigenvalue problem can be solved in general.

2.4. Gyroscopic Free System with 2 DOF

The following system constitutes a strongly simplified model of a gyroscopic system as it appears in rotordynamics. It consists of a disk rotating with constant speed and carrying a mass that can move in two directions (x_R, y_R) relative to the disk and that is attached to it by two springs:

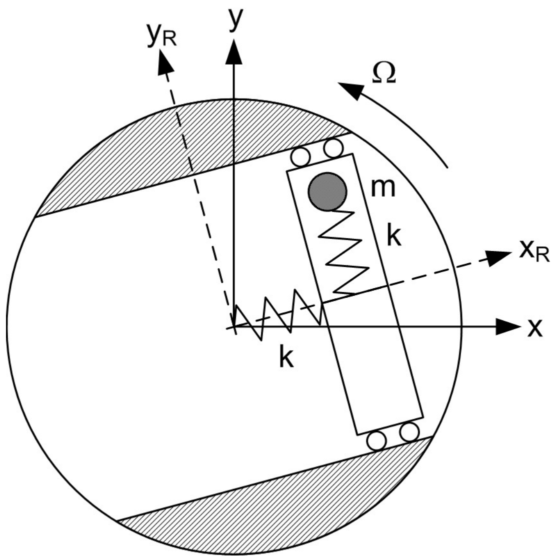

Figure 2.3: Simple gyroscopic system

The differential matrix equation can quite easily be derived in the rotating reference frame, i.e. if coordinates x_R and y_R are used:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{Bmatrix} \ddot{x}_R \\ \ddot{y}_R \end{Bmatrix} + \begin{bmatrix} 0 & -2m\Omega \\ 2m\Omega & 0 \end{bmatrix} \begin{Bmatrix} \dot{x}_R \\ \dot{y}_R \end{Bmatrix} + \begin{bmatrix} k - m\Omega^2 & 0 \\ 0 & k - m\Omega^2 \end{bmatrix} \begin{Bmatrix} x_R \\ y_R \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{2.16}$$

As we can already see the structure of this matrix DE is a bit different from what we have encountered before. Especially the “damping” matrix is not symmetric and additionally depends on the rotational speed Ω. We call such as system with a skew-symmetric speed dependent term a “gyroscopic” system with the matrix **G** containing the gyroscopic terms. Consequently, the DE can be written as:

$$\boxed{\mathbf{M\ddot{x}} + \mathbf{G\dot{x}} + \mathbf{Kx} = \mathbf{0}; \qquad \mathbf{G} = -\mathbf{G}^T} \tag{2.17}$$

The eigenvalue problem for (2.17) is similar to the case with damping and cannot, in general, be solved analytically.

Note that, for a system to be gyroscopic it always needs an even number of degrees of freedom.

Remark:

The simple model used in this section is, strictly speaking, not gyroscopic – it only ***appears*** to be gyroscopic when rotating coordinates are used for its description. One can show that, after transformation of DE (2.16) into the inertial system (x, y) using ordinary coordinate transformation matrices, the gyroscopic matrix vanishes and a purely conservative system (**M**, **K**) results. However, it was the aim of this section to show that skew-symmetric speed dependent terms in a matrix DE can occur and that their physical origin is completely different from damping effects. Gyroscopic effects resulting in skew-symmetric matrices always occur in rotating machinery. We will study gyroscopic effects further below when discussing rotordynamics.

2.5. Generalization to n DOF (Multi-DOF System)

2.5.1. Equation of Motion of a Multi-DOF System

As we already assume very straightforwardly a dynamic mechanical system having more than 2 DOF and constant parameters (time invariant system) must also be describable by a matrix DE formulation as derived above. In fact, one can show that any such system follows the description:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{P}\dot{\mathbf{x}} + \mathbf{Q}\mathbf{x} = \mathbf{f}(t); \quad \mathbf{M},\mathbf{P},\mathbf{Q} \in R^{nxn} \tag{2.18}$$

In (2.18) **M** is the mass matrix of the system. For a system with n DOF the dimension of **M** is also n. Furthermore, a system description by choosing appropriate coordinates can always be found so that the **M** is symmetric ($\mathbf{M} = \mathbf{M}^T$) and positive definite.

The right hand side of (2.18) is a general force vector with a general time dependency, rendering the DE inhomogeneous. We have seen in chapter 1 that the analysis of the response to an external excitation force is very important. This, of course, also holds for multi-DOF systems.

The general matrices **P** and **Q** for the speed and displacement dependent terms in (2.18) can always be split up into their symmetric and skew-symmetric components (this is possible for any matrix). By doing this we can introduce the symmetric damping matrix **D** and the skew-symmetric gyroscopic matrix **G**. Moreover, we obtain a new matrix unknown until now: the so-called non-conservative matrix **N**, which is also skew-symmetric, whereas the symmetric part of **Q** is the well-known stiffness matrix **K**:

$$\mathbf{D} = \mathbf{D}^T = \frac{1}{2}\left(\mathbf{P} + \mathbf{P}^T\right); \quad \mathbf{G} = -\mathbf{G}^T = \frac{1}{2}\left(\mathbf{P} - \mathbf{P}^T\right); \quad \mathbf{D} + \mathbf{G} = \mathbf{P} \tag{2.19}$$

$$\mathbf{K} = \mathbf{K}^T = \frac{1}{2}\left(\mathbf{Q} + \mathbf{Q}^T\right); \quad \mathbf{N} = -\mathbf{N}^T = \frac{1}{2}\left(\mathbf{Q} - \mathbf{Q}^T\right); \quad \mathbf{K} + \mathbf{N} = \mathbf{Q} \tag{2.20}$$

With equivalences (2.19) and (2.20) the DE (2.18) can be brought into the standard description of any dynamic mechanical multi-DOF system:

$$\boxed{\mathbf{M}\ddot{\mathbf{x}} + (\mathbf{D} + \mathbf{G})\dot{\mathbf{x}} + (\mathbf{K} + \mathbf{N})\mathbf{x} = \mathbf{f}(t)} \tag{2.21}$$

As already mentioned above the mass matrix **M** is always positive definite. For many (but not all) systems this also holds for the damping and stiffness matrices **D** resp. **K**. The property “positive definite” is important when investigating stability aspects of such a system. This, however, is not subject of this short lecture course.

2.5.2. Solution of the Multi-DOF Eigenvalue Problem

As outlined above along with simple 1 and 2 DOF examples the analysis of mechanical vibration systems as given by (2.21) always leads to an eigenvalue problem by which the frequencies of oscillation and the corresponding mode shapes, which are unique to every system, can be derived. We have also seen that the eigenvalue problem always yields eigenvalue solutions twice as many as the number of DOF n. This directly follows from the fact that mechanical DE are always of 2nd order (Newton’s law). Since the system is real, its eigenvalues and eigenvectors usually show up as complex conjugate pairs, however, purely real eigenvalues and eigenvectors are possible as well.

As pointed out above it is, in general, not possible to find an analytical solution to the eigenvalue problem of a multi-DOF system. Moreover, we have seen in equation (2.15) that in the general case a classic eigenvalue problem, as e.g. given by (2.11), cannot be obtained on the basis of the matrix DE (2.21). This is basically due to the fact that the DE of a mechanical system are of 2nd order.

However, a special eigenvalue problem can be formulated for (2.21) if the problem is transferred in the so-called state space. The basic idea is to formulate a set of differential equations of 1st order rather than of 2nd order. This can be achieved by introducing the state vector **z** containing the displacement vector **x** and the velocity vector $\dot{\mathbf{x}}$:

$$\mathbf{z} = \begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix} \tag{2.22}$$

For the eigenvalue analysis we only need the homogeneous part of matrix DE (2.21). With the help of state vector **z**, equation (2.21) can be identically written as:

$$\dot{\mathbf{z}} = \begin{Bmatrix} \dot{\mathbf{x}} \\ \ddot{\mathbf{x}} \end{Bmatrix} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}(\mathbf{K}+\mathbf{N}) & -\mathbf{M}^{-1}(\mathbf{D}+\mathbf{G}) \end{bmatrix} \begin{Bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{Bmatrix} = \mathbf{A}\,\mathbf{z};\ \mathbf{A} \in R^{2nx2n} \tag{2.23}$$

The 2nd order matrix DE system (2.21) has now been transformed into a 1st order matrix DE for **z**. The dimension of matrix **A**, however, is 2n, hence, twice the dimension of the physical matrices (**M**, **K**, etc.). Note that the state space matrix **A** contains all the information about the mechanical system (**M**,**D**,**G**,**K,N** matrices), thus, equation (2.23) must yield the identical eigenvalues λ as the original system (2.21). These eigenvalues can now be found by setting up the well-known guess function for z:

$$\boxed{\mathbf{z} = \tilde{\mathbf{z}} e^{\lambda t} \rightarrow [\lambda \mathbf{I} - \mathbf{A}]\tilde{\mathbf{z}} = \mathbf{0}} \tag{2.24}$$

Matrix equation (2.24) is the classic special eigenvalue problem. The 2n eigenvalues λ of **A** and the corresponding eigenvectors $\tilde{\mathbf{z}}$ can be found numerically by means of any matrix computation software (e.g. by MATLAB®). Note again that these eigenvalues λ are identical to the eigenvalues of (2.21). As **z** contains the mechanical system's vibration amplitudes and its velocities (expression (2.22)), the mode shapes of the original mechanical system can be easily obtained.

2.5.3. Time Response of a Multi-DOF System

Once, the eigenvalue analysis has been carried out the time response of the homogeneous system (no external excitation) can be obtained by a superposition of all eigenmodes oscillating with their eigenfrequencies. Completely analogously to (1.19) we obtain for the general vibration description:

$$\boxed{\mathbf{x}(t) = \sum_{k=1}^{2n} C_k \tilde{\mathbf{x}}_k e^{\lambda_\kappa t}} \tag{2.25}$$

The (complex) constants C_k have to be determined from the initial conditions (t=0) of the oscillation.

2.5.4. Frequency Response of a Multi-DOF System

A most important and powerful analysis tool for any kind of vibration system is the frequency response. As shown in sections 1.4.2 and 1.4.3 the frequency response can be derived from the system's transfer function by setting the complex frequency variable s to $j\Omega$, which corresponds to examining responses to purely harmonic excitations. In the multi-dimensional case frequency responses also become multi-dimensional, i.e. instead of transfer functions so-called transfer matrices $\mathbf{G}(j\Omega)$ must be considered, whose elements $g_{ij}(j\Omega)$ describe the frequency response relationship from the j-th input of the system to its i-th output.

From equation (2.21) the frequency response matrix, i.e. the relationship between the multi-dimensional excitation **F** and the displacement vector **x** can be easily derived:

$$\boxed{\mathbf{G}(j\Omega) = \left[-\Omega^2\mathbf{M} + \mathbf{K} + \mathbf{N} + j\Omega(\mathbf{D} + \mathbf{G})\right]^{-1}} \tag{2.26}$$

The similarity between (2.26) and the 1 DOF result (1.34) can easily be seen.

It is very essential to note here that each element of the frequency response matrix $\mathbf{G}(j\Omega)$ contains information about the entire system dynamics, i.e. all eigenvalues of the system are contained in every single element g_{ij} of $\mathbf{G}(j\Omega)$, a property which is of great help for system identification based on measurements (mathematically, this can be explained by the fact that the inverse of a matrix involves its determinant, and that the determinant contains all eigenvalues). Hence, it can be enough to measure one single transfer function element from one input to one output of a multi-DOF system to visualize all system eigenfrequencies. Such a measurement will be very similar to the resonance plot given in figure 1.7 for the damped 1 DOF system, but will generally show up more than one resonance peak (eigenfrequencies ω_i). In figure 2.4 the frequency response (amplitude) of an arbitrary multi-DOF system is shown (element i-j):

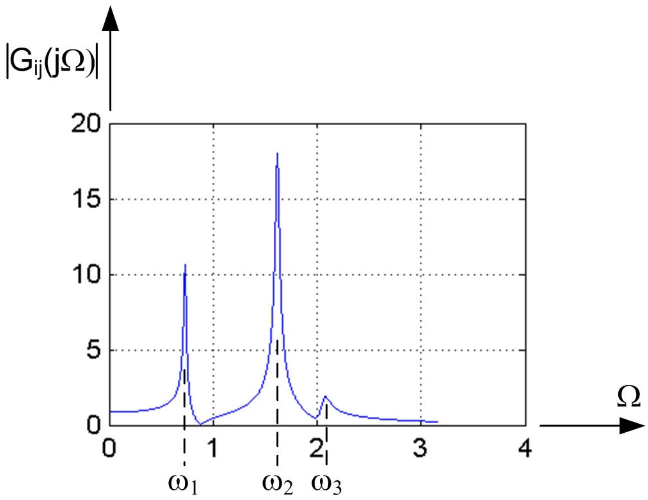

Figure 2.4: Multi-DOF frequency response element

The property of the full dynamic content in each element of the frequency response matrix **G**(jΩ) is practically used in the so-called "modal analysis" concept, a widely used approach for flexible structure identification. We will also make use of this concept further below when measuring transfer function elements by means of an active magnetic bearing (AMB) system in order to identify its bending modes.

2.6. Discretization of Continuous Systems, Finite Elements (FE)

As stated in the introduction to this chapter dynamic mechanical systems are so-called "continua" and, therefore, feature an infinite number of DOFs in reality. It is still possible to derive differential equations for such continua, however, these DE cannot be described by a set of linear 2nd order DEs with constant coefficients, as done before. In fact, the formulation of the equations of motion for continua always results in so-called "partial" differential equations, containing not only derivatives with respect to the time but also with respect to the physical coordinates involved. This can most easily be shown along with a simple example of an elastic beam fixed at its one end (figure 2.5):

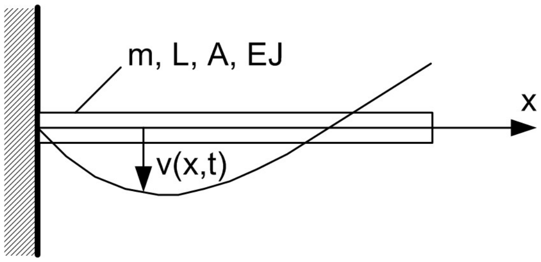

Figure 2.5: Elastic beam as a continuous mechanical vibration system

The derivation of the partial DE for the simple example is not shown here. Apart from the physical properties (mass m, length L, cross section A, bending stiffness EJ) the result involves partial derivatives of the displacement function v(x,t) with respect to the coordinate x (v', v'', etc.) and with respect to the time t ($\dot{v}$, $\ddot{v}$, etc.):

$$\boxed{\ddot{v} + \frac{L}{m} EJv'''' = 0} \tag{2.27}$$

A solution to (2.27) cannot be found analytically, however, it can be shown that, here again, an eigenvalue problem as shown in (2.24) will come up, from which the corresponding eigenfrequencies and mode shapes can be computed. Differing from multi-DOF systems this eigenvalue problem leads to a so-called transcendental equation that has to be solved numerically. The effort for doing this is already rather complicated for this very simple example. It is obvious that, for more elaborate and realistic continuous systems, complexity will even be higher.

Therefore, we must think of different methods to deal with "continuous" structures. One possibility is to virtually split up the system into parts where masses are concentrated and other mass-less parts where e.g. stiffnesses are concentrated. This approach ends up with a multi-DOF system description as introduced above (equation 2.21) involving matrices **M**, **D**, **G**, **K**, **N** for the discretized system, which can then by analyzed using the concepts shown above. However, it is often difficult to "decide" how to split up the system, i.e. how to carry out such a discretization manually.

A more theoretically founded method that leads to the same discrete multi-DOF system description (2.21) is the well known "finite element" (FE) approach. Today, FE modeling techniques have become very common since appropriate software packages and enough computing power are available. Compared to the FE approach other techniques such as "finite difference" modeling have, nowadays, become unimportant.

The theory of FE modeling cannot be looked at in detail here. However, the basic idea and concept can shortly be outlined.

The continuous structure is divided into sections, i.e. volume elements, where physical properties, such as mass, stiffness and geometric properties, do not change. For these elements the underlying DE must be known, hence must be available in the form of a multi-DOF system (2.21). This information is provided by the available elements within the FE software package. Element descriptions can also be found in literature. After definition of all elements, boundary conditions, initial conditions and external forces, the FE software package combines all single element descriptions into one global description by considering the known element dynamics as well as the transition conditions between the elements. This process ends up in setting up the global **M**, **D**, **G, K**, **N** matrices from the known basic element matrices. A sketch of this process is displayed in figure 2.6:

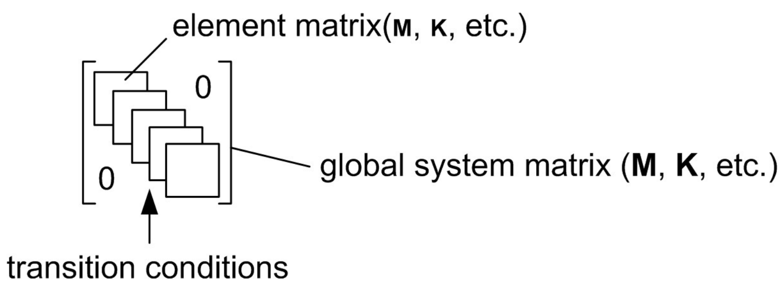

Figure 2.6: Finite element (FE) model setup process: from the local element matrix to the global system description matrix

FE modeling techniques are widely-used in the field of rotating machinery for the modeling of flexible rotors. For this case special axially symmetric FE rotor elements containing gyroscopic effects are used. Typical input data for each element of such a FE rotor model consists of the following information:

- geometric data (coordinates, diameters, element length)
- mass (density)
- stiffness data (Young's modulus, Poisson's ratio, etc.)
- additional inertia not contributing to stiffness (e.g. lumped masses)
- element couplings
- external forces

A typical example of FE modeling for rotor systems will be treated in the tutorial to this lecture course.

3. Rotordynamics of Active Magnetic Bearing (AMB) Systems

3.1. Introduction to Active Magnetic Bearing Systems

Active Magnetic Bearings (AMBs) can completely support a body by magnetic forces without any mechanical contact. Of technical interest are, in a first place, the ferromagnetic forces, which can be generated by permanent magnets or by actively controlled electromagnets. It has been known for a long time, however, that stable contact-free suspension in all degrees of freedom (DOFs) cannot be achieved by permanent magnets only, at least one active element is always necessary for stabilization of such a system. Magnetic bearings based on permanent magnets are passive elements unable to control rotor vibration actively. Passive systems are not covered in this short course.

Active systems were built as early as 1938 (Kemper, Germany) for experiments and later for momentum wheels in space applications. Due to the enormous progress achieved in electronics, the number of industrial applications in various fields has considerably increased during the last 20 years. A survey can be found in the literature, especially in the Proceedings of the International Symposium on Magnetic Bearings (ISMB).

Thanks to their physical principle, magnetic bearings have some unique and very interesting properties.

- Magnetic bearings work without any mechanical contact. Therefore, the bearings will have a long life with much reduced maintenance and with low bearing losses. Since no lubrication is required, processes will not be contaminated, which constitutes another important advantage over conventional bearing technologies. AMB systems can also work in harsh environments or in a vacuum.

- The reduced maintenance and the possibility for omission of the complete lubrication system lead to considerable cost reductions.
- The rotational speed is only limited by the strength of the rotor material (centrifugal forces). Peripheral speeds of 300 m/s are a standard in state-of-the-art AMB applications, a value not reachable by most other bearings.
- The electromagnetic bearing is an active element which enables accurate shaft positioning and which makes its integration into process control very easy. The vibrations of a rotor can be actively damped. It is also possible to let the rotor rotate about its principal axis of inertia to cancel the dynamic forces caused by the unbalance.
- Due to their built-in sensors and actuators AMB systems are perfectly suited for not only positioning and levitation of a rotor but also for serving additional purposes such as monitoring, preventive maintenance or system identification. These important features are possible without the need for any additional instrumentation.

Following from the mentioned features it can be found that an AMB system is a typical mechatronic product including a mechanical system part (rotor), a sensor, an actuator and a controller providing the AMB system with some level of “artificial intelligence”.

3.2. The Functional Principle

3.2.1. A Simple 1 DOF AMB System

The basic functional principle of an AMB can be briefly described as follows (figure 3.1).

The system itself is inherently unstable. This instability is caused by the attractive forces of the electromagnets. Therefore, active control of the magnets is necessary. For this. a sensor measures the displacement x of the supported rotor. A controller, nowadays most often a digital controller on the basis of a signal processor or microprocessor, uses the sensor information to derive an appropriate control signal u. This control signal is amplified by a power amplifier to drive the control current in the coil. The coil current together with the ferromagnetic material in the path of the coil causes a magnetic force to act on the rotor.

The electromagnetic force has to be calculated by the controller in such a way, that the rotor remains in its predefined and stable hovering position. Basically, the control operates in such a way that, when the rotor moves down, the sensor produces a displacement signal which leads to an increase in the control current. The increasing electromagnetic force then pulls the rotor back to its nominal position.

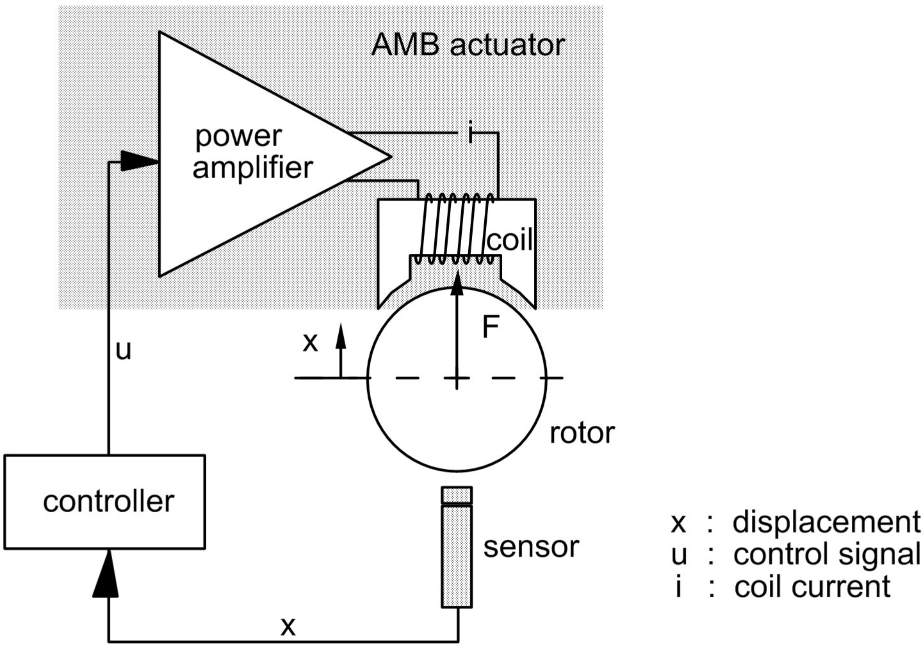

Figure 3.1: Basic principle of an active magnetic bearing (AMB)

3.2.2. Technical Realization of an AMB System

It is clear that a technical realization of the principle mentioned above, hence the levitation of a (rigid) rotor with six DOF, needs several bearing actuators. Most often they have to be interconnected by a multi-variable control. Figure 3.2 shows an example of a rotor assembly completely supported by two radial bearings and one thrust bearing. Therefore, five degrees of freedom have to be controlled (the 6^{th} DOF of the rigid body is the angular rotor position and is usually controlled by a motor).

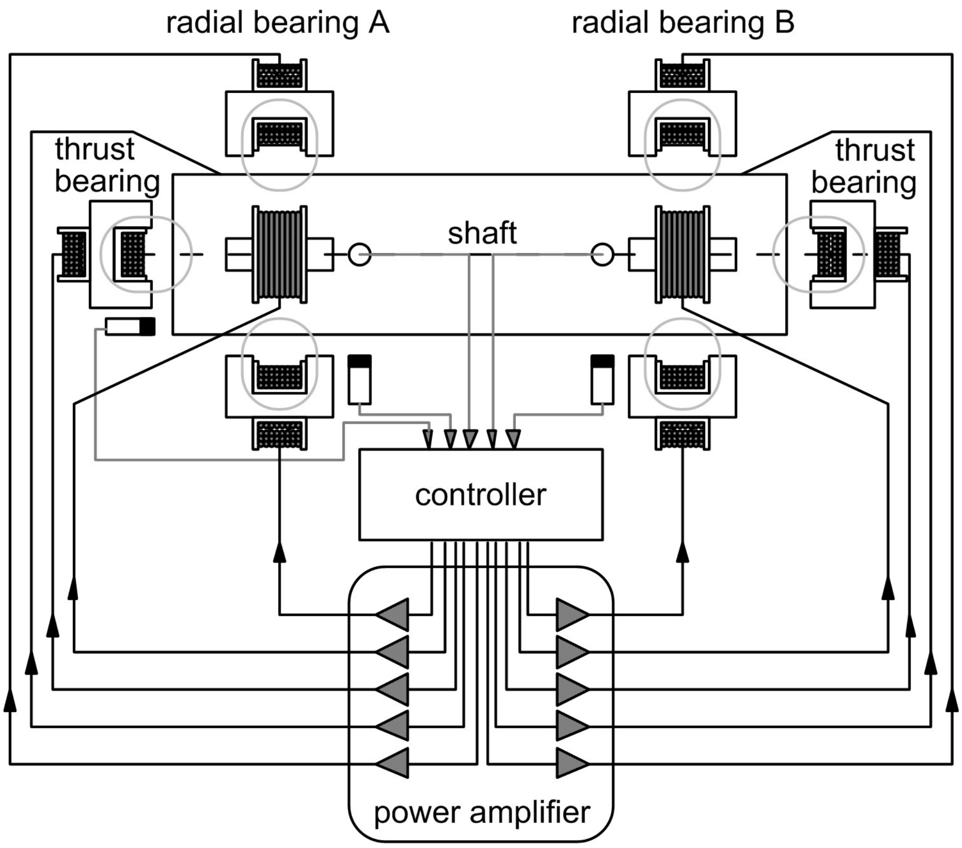

Figure 3.2: Technical realization of an AMB rotor system

Electromagnets:

It is important to see that, for each actuator channel, usually a pair of magnets is necessary, since electromagnets can only exert pulling (attractive) forces (in the example of figure 3.1 the rotor weight was used instead of an opposite magnet for “generating” a force opposite to the upper magnet, an approach which is only possible for specially configured systems). Consequently, a radial bearing most often consists of four bearing magnets, special configurations only using three magnets for two DOF are possible as well.

In general, AMB stators look rather similar to the stators of a motor, i.e. they consist of a stack of laminated soft-magnetic iron sheets. There are also attempts to combine the motor and bearing function in one single actuator. Such an approach is feasible for simple applications where the performance losses due to the need for comprising on both motor and bearing function are not important.

Sensors:

The sensors of an AMB system must measure the rotor position in a contact-free manner. In most applications eddy current or inductive sensors are used. Other possibilities include optical sensors, flux density sensors or a direct calculation of the position by using the current and voltage signals in the coils (so-called “self-sensing” bearing”).

Controller:

The controller hardware can be built either in analog or in digital technique. Because of their great flexibility, digital controllers have become more interesting recently. For high dynamics, µPs in multi-processor arrangements or DSPs are used.

The control signals generally depend on each other, i.e. each bearing force will depend on all sensor signals (so-called “centralized” or “Multiple Input Multiple Output” (MIMO) control). In some cases it is possible to control a pair of electromagnets only from their neighboring sensor (so-called “decentralized” or “Single Input Single Output” (SISO) control). If a SISO PID control scheme is applied to such a system, a pair of electromagnets will behave very similarly to a conventional spring damper element, with the advantage, though, of generating the forces in a contact-free manner and of being able to adapt the bearing characteristics any time to the operating requirements.

State-of-the-art control design techniques are PD, PID, optimal output feedback, observer based state feedback, μ synthesis and H^∞ approaches (MIMO and SISO). The axial control is usually decoupled from the radial channels and can be designed separately (SISO).

Power Amplifier:

Current power amplifiers in either analog (linear) or switched technology are used. For high power applications, switched amplifiers are preferred on account of their lower losses. Here again, AMB power amplifiers show a high level of similarity to motor controllers.

3.3. System Dynamics

As mentioned in chapter 2 the mechanical part of an AMB system, in most cases the levitated rotor, can be modeled as a so-called **M**,**D**,**G**,**K**,**N** system (see equation 2.21). The dimension of the matrices depends on the modeling approach (rigid body model, FE model, etc.). For the AMB controller design it is, in most cases, necessary to include some of the lower frequency flexible modes in the model, especially when so-called super-critical systems are considered where the rotational speed is above the eigenfrequencies of one or several bending modes.

In general, also the dynamics of the sensors, actuators and power amplifiers have to be considered. These systems are non-mechanical and, therefore, cannot be described by equation (2.21). However, a description of the system dynamics of such systems is always possible in the state space. Consequently, all system components, including the mechanical part, will be described by a state space description as shown in equation (2.23), augmented by the elements of the non-mechanical parts of the system.

In this lecture course we only focus on the mechanical part of the plant, hence, we assume that the dynamics of the other system elements are not important, i.e. that their typical time constants are much smaller than those of the mechanical plant. For many systems this simplification can be made without loss of important information.

3.3.1. Zero Speed AMB Rotor Model

In the following, for simplicity, emphasis is put on the investigation of the dynamic effects resulting from the radial motions of an AMB rotor. As mentioned, the axial motion is completely decoupled from all other motions and, correspondingly, this part of the dynamics can be modeled as a simple SISO system not being of particular interest here.

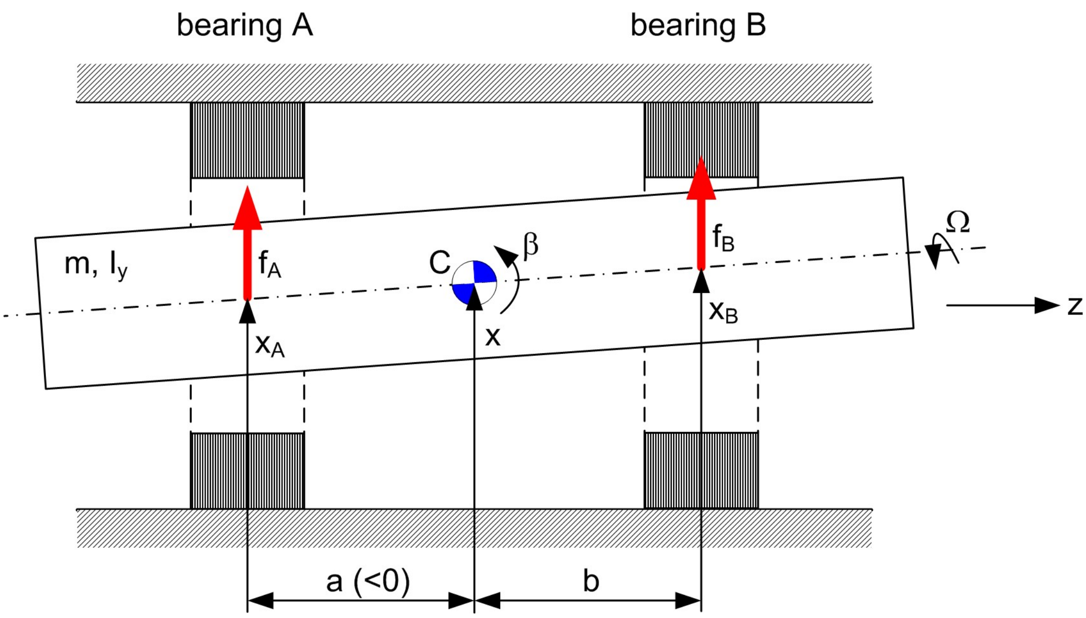

Figure 3.3: Simple rigid rotor model (beam model)

In figure 3.3 a simple rigid body rotor in AMBs is displayed. In a first step we only consider the rotor motion in one plane (x-z plane). It is obvious that this system only has 2 DOF, e.g. the position measured in each bearing. Therefore, the model description could be based on the two bearing resp. sensor positions x_A and x_B. For the subsequent analysis, however, it is more suitable to consider two other displacement signals, the position of the rotor's center of gravity C and the tilting angle β about the y axis. The transition between both coordinate systems resp. both model descriptions can be made by appropriate coordinate transformation matrices.

When using the center of gravity coordinates, system description becomes very simple and involves the rotor mass m and the rotor's moment of inertia I_y about the transverse axis y (Newton's and Euler's laws used for derivation of the matrix DE). Note that geometry parameters used in the model are sign-extended (length a is usually a negative quantity since the A bearing position has a negative z coordinate).

$$\begin{bmatrix} m & 0 \\ 0 & I_y \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\beta} \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ a & b \end{bmatrix} \begin{Bmatrix} f_A \\ f_B \end{Bmatrix} \tag{3.1}$$

Expression (3.1) is not intuitively recognizable as being of the type given by (2.21). However, when the concept of feedback control is introduced, the forces f_A and f_B become functions of the displacements x resp. β and of their derivatives, hence, the force term on the right hand side of equation (3.1) can be moved to the left hand side. For the example of a very simple decentralized PD feedback, where the forces are a linear combination of the displacements and velocities in the bearings, equation (3.1) can be brought into the general form corresponding to (2.21):

$$f_A = -k_A x_A - d_A \dot{x}_A \; ; \qquad f_B = -k_B x_B - d_B \dot{x}_B \tag{3.2}$$

$$\begin{Bmatrix} x \\ \beta \end{Bmatrix} = \frac{1}{b-a} \begin{bmatrix} b & -a \\ -1 & 1 \end{bmatrix} \begin{Bmatrix} x_A \\ x_B \end{Bmatrix} \leftrightarrow \begin{Bmatrix} x_A \\ x_B \end{Bmatrix} = \begin{bmatrix} 1 & a \\ 1 & b \end{bmatrix} \begin{Bmatrix} x \\ \beta \end{Bmatrix} \tag{3.3}$$

$$\begin{bmatrix} m & 0 \\ 0 & I_y \end{bmatrix} \begin{Bmatrix} \ddot{x} \\ \ddot{\beta} \end{Bmatrix} + \begin{bmatrix} d_A + d_B & d_A a + d_B b \\ d_A a + d_B b & d_A a^2 + d_B b^2 \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{\beta} \end{Bmatrix}$$

$$+ \begin{bmatrix} k_A + k_B & k_A a + k_B b \\ k_A a + k_B b & k_A a^2 + k_B b^2 \end{bmatrix} \begin{Bmatrix} x \\ \beta \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{3.4}$$

Hence, for such a decentralized feedback the system becomes of the known type of a 2 DOF system with mass, damping and stiffness matrices **M**, **D**, **K**:

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{D}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{0} \; ; \quad \mathbf{x} = \begin{Bmatrix} x \\ \beta \end{Bmatrix} \tag{3.5}$$

It is obvious that the rotor motion description in the perpendicular y-z plane must be identical, since the rotor is symmetric:

$$\mathbf{M}\ddot{\mathbf{y}} + \mathbf{D}\dot{\mathbf{y}} + \mathbf{K}\mathbf{y} = \mathbf{0} \; ; \quad \mathbf{y} = \begin{Bmatrix} y \\ -\alpha \end{Bmatrix} \tag{3.6}$$

The reason why the tilting angle α comes in with a negative sign is simply due to the coordinate system's rotation direction convention: A positive rotation β about the y axis produces a positive displacement in x direction for a positive z coordinate, whereas a corresponding rotation α produces a negative displacement in y direction for the same positive z coordinate. By introducing the negative sign for the tilting angle α, however, it can be achieved that the sub matrices **M**, **D**, **K** are identical in both motion planes, something which certainly makes sense for the description of a symmetric system. The overall 4 DOF system description now becomes:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}\ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix}\dot{\mathbf{q}} + \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix}\mathbf{q} = \mathbf{0} \; ; \quad \mathbf{q} = \begin{Bmatrix} x \\ \beta \\ y \\ -\alpha \end{Bmatrix} \tag{3.7}$$

3.3.2. Gyroscopic AMB Rotor Model

From equation (3.7) we can guess that the sub systems (motion in x-z plane resp. y-z plane) are completely decoupled or, in other words, any disturbance in x direction will not generate any displacement in y direction and vice versa. However, it turns out that this is only true if the rotor is at stand still. For the rotating system a very important model augmentation has to be carried out by considering the momentum vector **L**.

In figure 3.4 the instantaneous motion state of a rigid rotor is displayed. The rotor rotates about the z axis and, hence, its momentum **L**(t) is the product of its moment of inertia I_z about the rotation axis and the rotation speed Ω. The momentum vector points in z direction (direction of rotation). At the time t an external moment of force $\mathbf{M}_x$(t) is applied in x direction. Following Euler's law the momentum change **dL** is proportional to the moment of force and has the same direction as this moment of force. Hence, the change of the momentum **dL** also points in x direction. Consequently, all material elements on the rotation axis will move in positive or negative x direction (except for the center of gravity that does not change position in this case). This motion, finally, corresponds to a tilting about the y axis with tilting angle β. What we would have intuitively expected, however, is a tilting of the rotor about the x axis, since the external moment of force is also applied about the x axis.

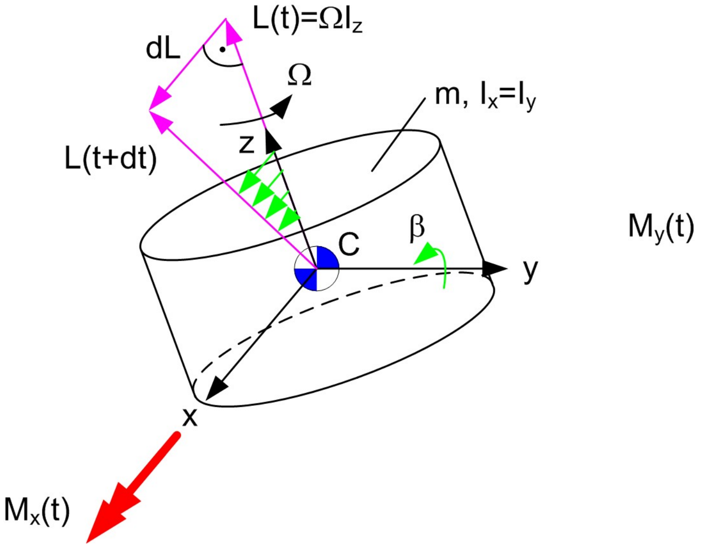

Figure 3.4: Momentum and its change due to a moment of force for a rigid rotor

A displacement reaction perpendicular to the applied external disturbance is an effect well-known from the behavior of a spinning gyroscope. Therefore, the effect is also called the "gyroscopic" effect. The fundamental result from it is that the rotor motions in the x-z and y-z planes become coupled as soon as tilting is involved.

In this lecture course we are not going to carry out the detailed derivation of the gyroscopic coupling effect on the system's DE, only the result is presented here.

Due to the gyroscopic coupling an additional sub matrix **G** will appear in expression (3.7) that involves the rotor's moment of inertia I_z about the rotation axis and the rotational speed Ω:

$$\begin{bmatrix} \mathbf{M} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix} \ddot{\mathbf{q}} + \begin{bmatrix} \mathbf{0} & \mathbf{G} \\ -\mathbf{G} & \mathbf{0} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{D} & \mathbf{0} \\ \mathbf{0} & \mathbf{D} \end{bmatrix} \dot{\mathbf{q}} + \begin{bmatrix} \mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{K} \end{bmatrix} \mathbf{q} = \mathbf{0}\,; \quad \mathbf{G} = \begin{bmatrix} 0 & 0 \\ 0 & \Omega I_z \end{bmatrix} \tag{3.8}$$

As we can see a skew-symmetric matrix for the velocity term appears in equation (3.8). This matrix is the gyroscopic term also contained in (2.21) and first introduced in section 2.4. It is also seen that the sub matrix **G** couples the motions in the x-z and y-z planes (coupling of tilting angles α and β), which is exactly what we expected based on the simple phenomenological approach shown above.

Matrix DE (3.8) holds for the majority of all AMB systems. The additional introduction of non-conservative effects (**N** matrix) is not necessary in general.

DE equation (3.8) also holds for the case of flexible AMB rotor systems with more than 4 DOF. The dimension of the matrices will then depend on the complexity of the model (e.g. number of elements in case of FE modeling). A model reduction on the basis of known reduction techniques is always possible, however, one should be careful to include enough flexible modes in order to make the model representative, especially when having to pass through bending critical speeds.

Finally, it has to be made clear that FE modeling techniques, when based on suitable rotor elements, also provide the gyroscopic part of (3.8), for the rigid rotor part as well as for the flexible modes. Hence, a good FE model provides all the necessary information for a rotordynamic analysis of the rotating system.

3.4. Rotordynamic Analysis

This section is intended to shortly investigate the most important effects appearing in rotordynamics. Emphasis is put on effects that generally appear, not only in the case of AMB systems. Some unique rotordynamic features of AMBs, such as the unbalance force cancellation concept, are as well treated in the following sections.

No investigation of AMB system stability can be carried out here. AMB controller design and stability investigation are part of a more control theoretical focus, which cannot be covered here.

Apart from very special rotordynamic effects such as the destabilization due to inner damping (non-conservative forces), which can appear in some rare rotor systems, the most important phenomena are the dependence of the system eigenfrequencies on the rotational speed and the system response to unbalance excitation. These two effects are shortly discussed in the following.

3.4.1. Eigenfrequencies as a Function of the Rotational Speed

Based on the rotordynamic description (3.8) the system eigenvalues can immediately be calculated using equation (2.24) (state space). It is clear that each system will provide its particular eigenvalues and eigenmodes. However, there are some fundamental behavior elements common to all rotating systems, whether flexible or rigid, whether in AMBs or in conventional bearings.

From equation (3.8) we can judge that the only dependence on the rotational speed Ω is the skew-symmetric gyroscopic term. Hence, all eigenvalue dependencies on the rotor speed are solely coming from this term. Figure 3.5 displays these dependencies for a typical rotor system in a so-called “Campbell” diagram (2x4 DOF system):

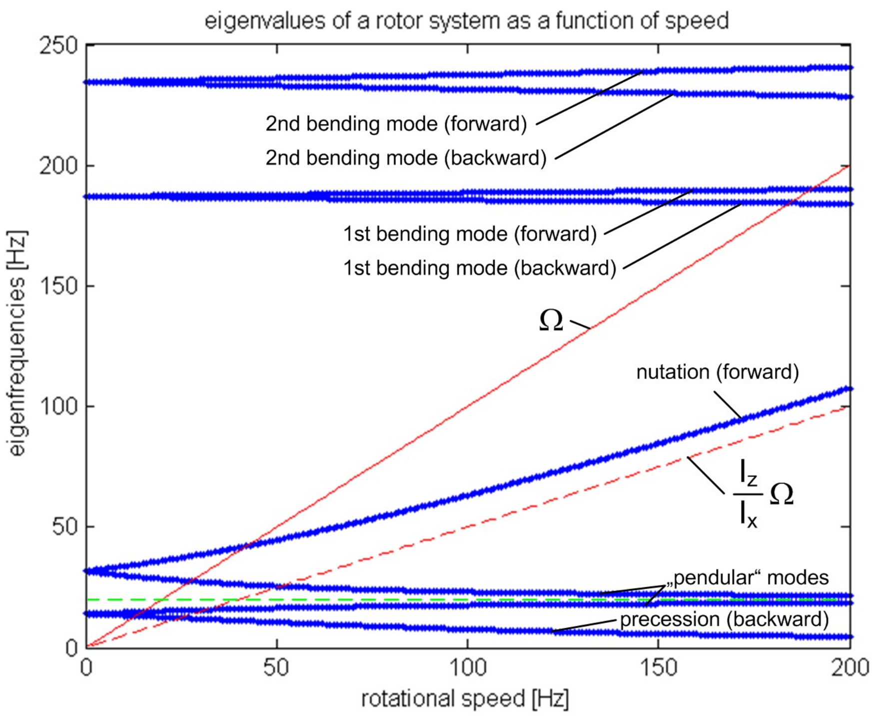

Figure 3.5: Campbell diagram of a typical flexible rotor system

Apart from the motion coupling, i.e. from the coupling of the eigenmodes in the x-z and y-z planes, the gyroscopic terms also effectuate a “split” of the eigenvalues depending on the rotational speed. At stand still the eigenvalues of any mode in the x-z plane are always identical to the corresponding mode in the y-z plane (symmetric rotor). Hence, there are totally 8 but only 4 different eigenfrequencies at stand still. However, with increasing speed, each pair of identical eigenfrequencies splits up into one eigenvalue that increases and another one that decreases with speed. The eigenmodes with increasing eigenvalue behavior are the so-called “forward” modes, since their motions have the same sense of rotation as the rotational speed itself. Correspondingly, all eigenmodes with decreasing eigenvalue dependency are the so-called “backward” modes.

Another general behavior can be attributed to the rigid body modes (low frequency). Their zero speed eigenvalues depend on the external stiffness of the system (bearing stiffness). When speed increases the two rigid body eigenmodes of each plane become coupled and show up as so-called “conical” and “pendular” modes. All modes exist as “forward” and “backward” modes. In the case of the conical modes they are called “nutation” (forward) and “precession” (backward) mode. The conical modes correspond to the modes of a pure gyroscope.

At high rotational speeds, the influence on the bearing stiffness vanishes for the conical modes and the eigenfrequencies become pure determined by the mass and gyroscopic terms. Whereas the precession mode’s eigenfrequency approaches zero, that of the nutation mode asymptotically reaches the linear function given by the ratio of the 2 moments of inertia I_x and I_z. From this knowledge one can always guess the frequency of the nutation of a rotor system at high rotation speeds, just based upon the ratio of the moments of inertia which can be obtained from a FE modeling process. For this no model dynamics analysis is necessary.

Whereas the conical modes (nutation and precession) are strongly influenced by the gyroscopic effects since tilting is involved, the pendular modes become less and less influenced by gyroscopic effects with increasing speed, hence, their mode shapes become pure translations of the center of gravity in the x-z and the y-z planes. Moreover, these motions become decoupled at high rotational speeds and the corresponding eigenfrequencies are determined by the mass and the external stiffness, but not by the gyroscopic terms. Hence, the behavior of the pendular modes is entirely different from that of the conical modes.

As mentioned above also the bending modes and their eigenfrequencies are influenced gyroscopically. The split up of the eigenfrequencies very much depends on the particular system. There is no general rule for asymptotic behaviors as for the rigid body modes.

It is obvious that the knowledge of the Campbell diagram is essential for AMB systems with respect to controller design. As plant eigenvalues change, the control must either be gain-scheduled with speed, or it must be robust enough to tolerate rotor speed induced plant changes. Moreover, mode shapes also change with speed and might, at a specific rotor speed, result in a configuration where a sensor is located in a so-called nodal point which features an inherently zero displacement amplitude. In this case AMB control can become heavily affected by the gyroscopic effects.

3.4.2. Unbalance Response

Another most important rotordynamic effect is the system's response to unbalance force excitation. Unbalances are present in every rotor system since perfect balancing is not possible or, if feasible at all, very costly, especially if balancing of the flexible modes has to be included. Another possibility for reducing the system's unbalance response is to increase the (external) damping, which, however, is difficult for conventional rotor bearing systems. For AMB systems, instead, such an approach is rather straightforward. More damping can be introduced by implementing an appropriate controller (see also example of damping bending criticals below). If there is only little or no damping present, unbalance responses can become very large and dangerous (resonance effect, see section 1.4.2).

In figure 3.6 unbalance response at an arbitrary measurement (sensor) point for the same system as discussed in the previous section is displayed for an assumed unbalance and for different external damping levels.

As can be clearly seen the resonance amplification strongly depends on the amount of damping, a property already found with the simplest possible 1 DOF system (see section 1.4.2).

Astonishing, however, is the finding that only 3 resonance peaks are visible, whereas, according to the Campbell diagram, there are 6 eigenfrequencies in the range of the rotational speed (forward/backward conical mode, forward/backward pendular mode and forward/backward 1st bending mode). This finding is an important property of every rotor system: As unbalance is a force excited by the rotation itself, it must be considered not only a harmonic force but a harmonic force with a given sense of rotation. This sense of rotation is always "forward" for the unbalance excitation and, hence, only the forward eigenmodes can be excited by the unbalance.

Note that, by any different kind of harmonic excitation, all the eigenmodes (forward and backward) can be excited. It is also essential to notice that a particular resonance peak might not be visible at all available measurement points. This corresponds to the "controllability" and "observability" properties known from control theory and can happen when a vibration node is located either at a sensor or at an actuator position.

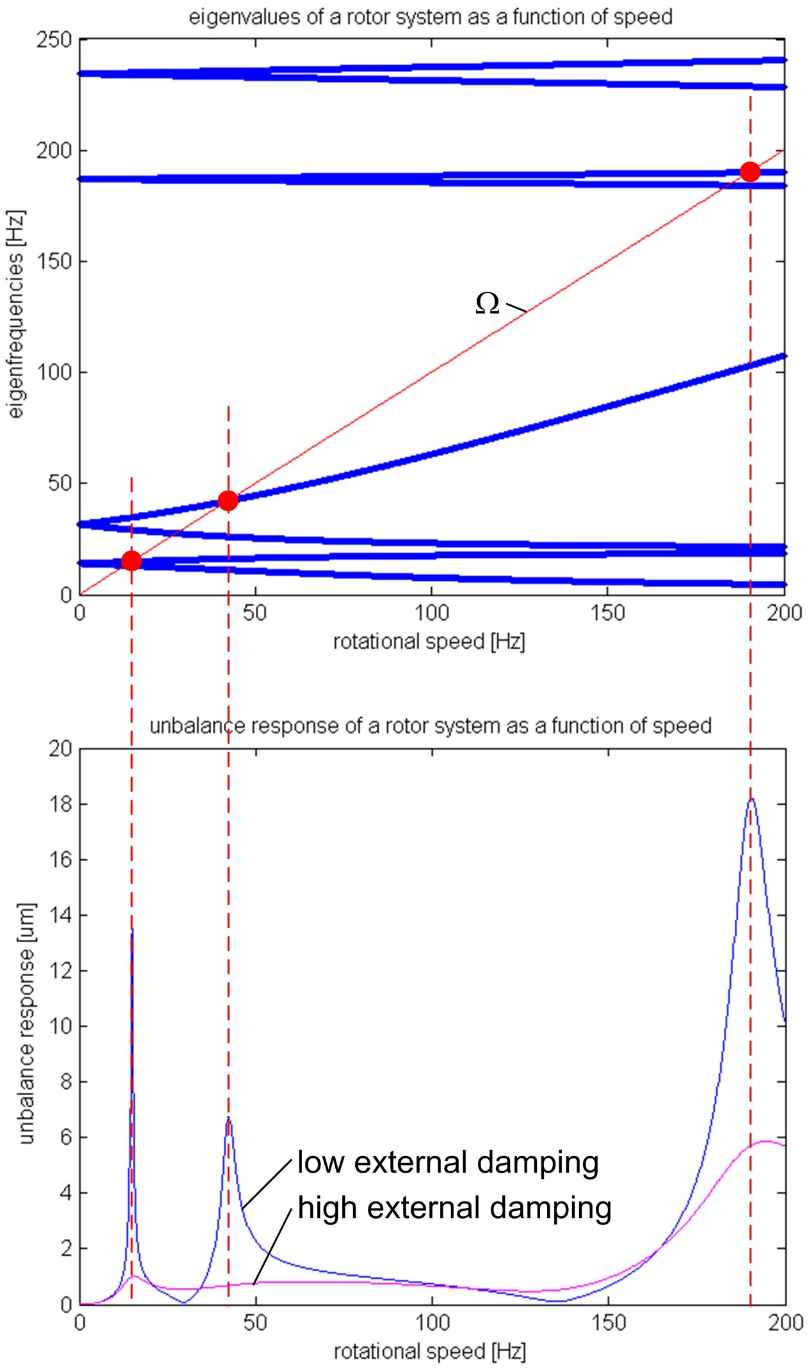

Figure 3.6: Unbalance response of a typical flexible rotor system

3.5. Special Features of AMB Rotor Systems

3.5.1. Infinite Static Stiffness

Active control allows for compensating of long term displacement deviations by using an integral feedback. By feeding back the integral of the sensor signal, the bearing force will keep increasing until there is no deviation between the desired and the actual rotor position, independently on the level of the external force (as long as the bearing's load capacity is not reached). This is often thought of as having an infinite static stiffness. Integral feedback compensates for static forces, as for example for the rotor weight. Without integral feedback the rotor weight leads to a displacement according to the stiffness given by the proportional term of the PD controller.

A PD controller with additional integral feedback is called a PID controller. For slow integration speed the integral feedback has a very limited interference with the proportional feedback and, consequently, the eigenvalues of the PD controlled system do not change much, however an additional eigenvalue for each integrator is added to the system.

The possibility of providing a virtually infinite static stiffness is an important advantage of AMBs over conventional bearing solutions.

3.5.2. Unbalance Force Rejection

Another most important advantage of AMB systems is their ability to adaptively cancel the effect of unbalance force on the machine vibration.

Unbalance excitation is a major concern in rotating machines. Since the unbalance forces are well correlated with the rotational speed, i.e. they have an amplitude and phase angle relative to the rotor which only depends on the rotational speed, their controller response can be eliminated by adding a suitable signal to the bearing system. A block diagram of such an unbalance force rejection scheme is shown in figure 3.7.

A so-called "Unbalance Force Rejection Control" (UFRC) block is added to the standard feedback loop of plant (rotor) and controller. The UFRC provides an extremely narrow band filtering of the controller input signal such that only the rotation speed synchronous harmonic component can pass. This component is subtracted from the original sensor signal such that the controller input is free of any synchronous signal component. Therefore, also the controller output is free of such components, which is equivalent to not generating any synchronous reaction forces. In other words: Despite of the existing unbalance the rotor can rotate force-free about its principal axis of inertia and no synchronous vibration forces are transmitted to the machine foundation.

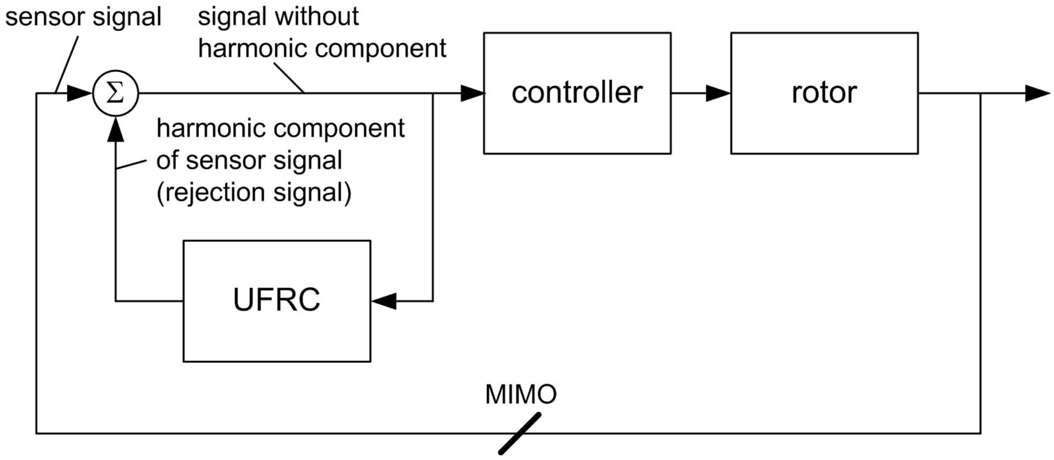

Figure 3.7: Unbalance force rejection control scheme (UFRC)

This unique property is only possible thanks to active control and thanks to the fact that AMBs operate without contact. With a mechanical contact given, forces would immediately be generated due to the non-zero rotor orbit.

UFRC is also usable within the rigid body critical speeds. If properly parameterized for overall system stability, it can theoretically operate down to zero speed. At zero speed UFRC is identical to the so-called “zero power” control concept.

The operation of UFRC is limited to suitably small unbalance levels, i.e. UFRC can only work as long as the rotor orbit is smaller than the touch down bearing gap. In practice, however, this limitation is never met, and most AMB rotors can be operated perfectly without any need for balancing at all. With conventional bearings, though, zero vibration force transmission is only possible by perfect balancing.

It is important to emphasize that UFRC does not carry out any balancing of the rotor. Instead, UFRC only allows for force-free rotation about the rotor’s principal axis of inertia in the presence of unbalances. However, in some literature this concept is also misleadingly called “ABS” (“Automatic Balancing System”).

3.5.3. Active Vibration Control in Bending Criticals

A variation of UFRC can be used for passing critical speeds. While, for a realistic rotor with very small inner damping, it is physically not possible to pass bending critical speeds (resonances) in a force-free way, i.e. with a pure UFRC scheme, UFRC can provide another purely harmonic and synchronous output signal that serves for generating an active damping force at the controller output. A corresponding block diagram is shown in figure 3.8.

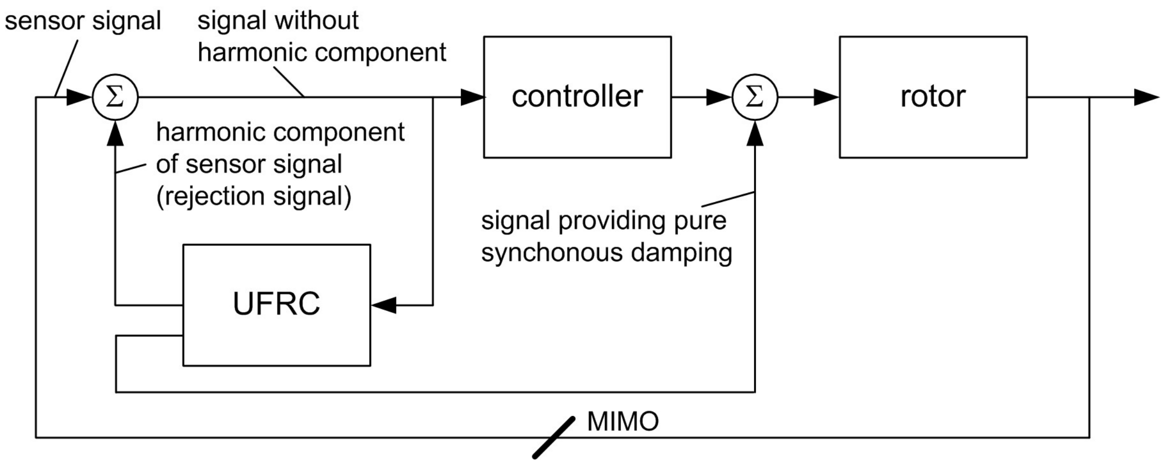

Figure 3.8: Adaptation of UFRC for critical speed damping

Note that the necessary damping for passing a bending critical is not achieved by the standard controller, whose synchronous signal remains cancelled by the conventional UFRC scheme at its input. Instead, the damping force is solely generated by the 2nd output signal of the UFRC block. In figure 3.9 the effect of synchronous damping on the vibration amplitude is shown.

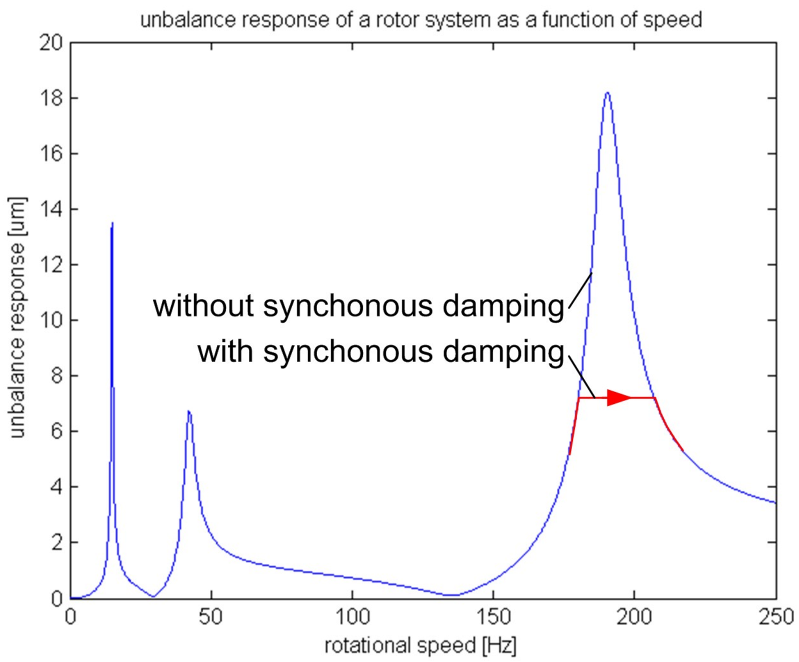

Figure 3.9: Synchronous damping effect on the critical speed amplitude

3.5.4. Built-In Instrumentation for System Identification and Monitoring

As already mentioned AMBs are not only an interesting bearing device with actively adjustable characteristics, but they can also serve as an excellent monitoring system. The displacement sensors in cooperation with suitable controller functionalities allow for measurement and identification of the behavior of the rotor system. In addition to its levitation function the magnetic bearing actuators can be used as an excitation device for driving the rotor into a desired state, e.g. into a steady-state harmonic vibration with a given amplitude and a predefinable frequency. This concept directly allows for the on-line measurement of system frequency responses, without the need for any additional instrumentation and without having to carry out any hardware adjustments for measurement signal access.

Appropriate AMB controllers can provide the following monitoring and diagnostic tools:

- on-line displacement monitoring
- on-line dynamic force measurement and monitoring
- on-line unbalance measurement and monitoring
- identification and monitoring of process forces
- identification of critical speeds
- system identification (eigenvalues, mode shapes)
- step response, harmonic perturbation response, etc.

If the controller is based on a microprocessor system most of the monitoring and diagnostic features can be provided by software packages, without the need for additional hardware. For doing this, suitable excitation input points have to be defined in the digital feedback control architecture. Moreover, signal processing algorithms such as an FFT have to be implemented on the AMB controller. In figure 3.10 possible excitation points in an AMB control loop are shown:

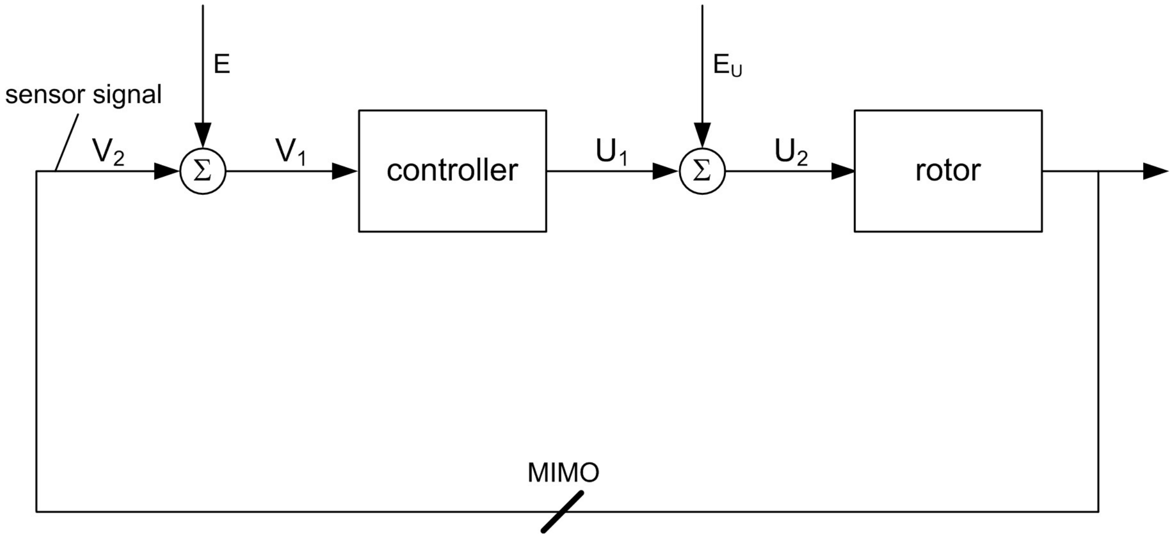

Figure 3.10: AMB feedback loop with additional excitation inputs for system identification and monitoring

By means of excitation inputs such as E resp. E_U (see figure 3.10) and by suitable signal processing algorithms and communication tools virtually any transfer function from any input to any output signal can be directly measured on the AMB system and then transferred to a host computer for further analysis and display. Moreover, it is possible to measure SISO or MIMO transfer functions. Hence, a complete transfer function $G(j\Omega)$ as given by equation (2.26) can be directly measured on-line and compared with the result obtained from the FE model introduced in equation (2.26). The comparison result can be used for updating the FE model, e.g. for adjusting the eigenfrequencies. A better model can then serve as a basis for an improved controller design.

Apart from this plant identification possibility other important system properties such as the sensitivity function or the complementary sensitivity function for verifying the controller performance and robustness can be easily obtained by suitable on-line measurements, a unique feature which is extremely important during the controller tuning phase of AMB systems.

Finally, effects resulting from non-modeled system dynamic components can be easily recognized in the measurements and can help in the optimization of the overall system performance.

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