Exercise B. Indefinite Integrals

Find the indefinite integrals by inspection using the differentiation table, the integration table, and the basic properties of integrals.

B1. $\int x^5\,\mathrm{d}x$ Ans: $x^6/6 + C$

B2. $\int 5x^4\,\mathrm{d}x$ Ans: $x^5 + C$

B3. $\int (3 + 3\sqrt{x})\,\mathrm{d}x$ Ans: $3x + 2x^{3/2} + C$

B4. $\int \dfrac{1}{x^3}\,\mathrm{d}x$ Ans: $-1/(2x^2) + C$

B5. $\int (2 + x^3)\,\mathrm{d}x$ Ans: $2x + x^4/4 + C$

B6. $\int (3 - x + 2x^2)\,\mathrm{d}x$ Ans: $3x - x^2/2 + 2x^3/3 + C$

B7. $\int (1 + x + x^2 + \cdots + x^{n-1})\,\mathrm{d}x$ Ans: $x + x^2/2 + x^3/3 + \cdots + x^n/n + C$

B8. $\int (3 - \cos x + 3x^2)\,\mathrm{d}x$ Ans: $3x - \sin x + x^3 + C$

B9. $\int (\sec^2 x + 5\sin x)\,\mathrm{d}x$ Ans: $\tan x - 5\cos x + C$

B10. $\int (x + 1/x)\,\mathrm{d}x$ Ans: $x^2/2 + \ln|x| + C$

B11. $\int (2\csc^2 x - 3/x)\,\mathrm{d}x$ Ans: $-2\cot x - 3\ln|x| + C$

B12. $\int (2\mathrm{e}^x - 3/x^2)\,\mathrm{d}x$ Ans: $2\mathrm{e}^x + 3/x + C$

B13. $\int \dfrac{2}{\sqrt{4 - x^2}}\,\mathrm{d}x$ Ans: $2\sin^{-1}(x/2) + C$

B14. $\int \dfrac{2}{9 + x^2}\,\mathrm{d}x$ Ans: $\frac{2}{3}\tan^{-1}(x/3) + C$

B15. $\int (2x + 3)^3\,\mathrm{d}x$ Ans: $\frac{1}{8}(2x + 3)^4 + C$

B16. $\int \cos(2x + 3)\,\mathrm{d}x$ Ans: $\frac{1}{2}\sin(2x + 3) + C$

B17. $\int \mathrm{e}^{3x}\,\mathrm{d}x$ Ans: $\frac{1}{3}\mathrm{e}^{3x} + C$

B18. $\int \sec(2x - 3)\tan(2x - 3)\,\mathrm{d}x$ Ans: $\frac{1}{2}\sec(2x - 3) + C$

B19. $\int \sec^2(2x + 3)\,\mathrm{d}x$ Ans: $\frac{1}{2}\tan(2x + 3) + C$

B20. $\displaystyle\int \sec(2x)\,\mathrm{d}x$ Ans: $\frac{1}{2}\ln|\sec(2x)+\tan(2x)|+C$

B21. $\displaystyle\int \frac{1}{5-3x}\mathrm{d}x$ Ans: $-\frac{1}{3}\ln|5-3x|+C$

B22. $\displaystyle\int \tan(5x+3)\,\mathrm{d}x$ Ans: $\frac{1}{5}\ln|\sec(5x+3)|+C$

B23. $\displaystyle\int \frac{2x}{3+x^2}\,\mathrm{d}x$ Ans: $\ln|x^2+3|+C$

B24. $\displaystyle\int \frac{\cos x}{\sin x+3}\,\mathrm{d}x$ Ans: $\ln|\sin x+3|+C$

B25. $\displaystyle\int \frac{\mathrm{e}^{2x}}{\mathrm{e}^{2x}-5}\,\mathrm{d}x$ Ans: $\frac{1}{2}\ln|\mathrm{e}^{2x}-5|+C$

Substitution

B26. $\displaystyle\int 2\cos^3 x\sin x\,\mathrm{d}x$ Ans: $-\frac{1}{2}\cos^4 x+C$

B27. $\displaystyle\int \frac{\cos x}{\sin^3 x}\,\mathrm{d}x$ Ans: $-1/(2\sin^2 x)+C$

B28. $\displaystyle\int x(x^2+2)^3\,\mathrm{d}x$ Ans: $\frac{1}{8}(x^2+2)^4+C$

B29. $\displaystyle\int \frac{x\,\mathrm{d}x}{(x^2+2)^2}$ Ans: $\dfrac{-1}{2(x^2+2)}+C$

B30. $\displaystyle\int x^3(x^4+1)^5\,\mathrm{d}x$ Ans: $\frac{1}{24}(x^4+1)^6+C$

B31. $\displaystyle\int \frac{\ln x}{x}\,\mathrm{d}x$ Ans: $\frac{1}{2}(\ln x)^2+C$

B32. $\displaystyle\int \frac{\mathrm{d}x}{x\ln x}$ Ans: $\ln\ln x+C$

B33. $\displaystyle\int \frac{\tan^{-1}x}{1+x^2}\,\mathrm{d}x$ Ans: $\frac{1}{2}(\tan^{-1}x)^2+C$

B34. $\displaystyle\int \cos x\exp(\sin x)\,\mathrm{d}x$ Ans: $\exp(\sin x)+C$

B35. $\displaystyle\int (3x+2)^{10}\,\mathrm{d}x$ Ans: $\frac{1}{33}(3x+2)^{11}+C$

B36. $\displaystyle\int x^3(x^2+1)^n\,\mathrm{d}x$ Ans: $\dfrac{(x^2+1)^{n+2}}{2(n+2)}-\dfrac{(x^2+1)^{n+1}}{2(n+1)}+C$

B37. $\displaystyle\int x(x+1)^{95}\,\mathrm{d}x$ Ans: $\frac{1}{97}(x+1)^{97}-\frac{1}{96}(x+1)^{96}+C$

B38. $\displaystyle\int x\sqrt{2x+3}\,\mathrm{d}x$ Ans: $\frac{1}{10}(2x+3)^{5/2}-\frac{1}{2}(2x+3)^{3/2}+C$

Products of sines and cosines

B39. $\int 2\cos x \sin x \, \mathrm{d}x$ Ans: $\sin^2 x + C$

B40. $\int 2\sin 3x \cos 2x \, \mathrm{d}x$ Ans: $-\frac{1}{5}\cos 5x - \cos x + C$

B41. $\int 14\cos 3x \cos 4x \, \mathrm{d}x$ Ans: $\sin 7x + 7\sin x + C$

B42. $\int 8\sin 3x \sin x \, \mathrm{d}x$ Ans: $-\sin 4x + 2\sin 2x + C$

B43. $\int 4\sin^2 x \, \mathrm{d}x$ Ans: $2x - \sin 2x + C$

B44. $\int 4\cos^2 x \, \mathrm{d}x$ Ans: $2x + \sin 2x + C$

B45. $\int 3\sin^2 x \cos x \, \mathrm{d}x$ Ans: $\sin^3 x + C$

B46. $\int 3\sin x \cos^2 x \, \mathrm{d}x$ Ans: $-\cos^3 x + C$

B47. $\int 32\sin^2 x \cos^2 x \, \mathrm{d}x$ Ans: $4x - \sin 4x + C$

B48. $\int 12\sin^3 x \, \mathrm{d}x$ Ans: $-9\cos x + \cos 3x + C$

Integration by parts

B49. $\int x\sin 3x \, \mathrm{d}x$ Ans: $-\dfrac{x\cos 3x}{3} + \dfrac{\sin 3x}{9} + C$

B50. $\int x\cos bx \, \mathrm{d}x, \quad (b \neq 0)$ Ans: $\dfrac{x\sin bx}{b} + \dfrac{\cos bx}{b^2} + C$

B51. $\int x\ln x \, \mathrm{d}x$ Ans: $\dfrac{x^2\ln x}{2} - \dfrac{x^2}{4} + C$

B52. $\int x\mathrm{e}^{2x} \, \mathrm{d}x$ Ans: $\mathrm{e}^{2x}(2x-1)/4 + C$

B53. $\int \ln(3x) \, \mathrm{d}x \quad (x > 0)$ Ans: $x\ln(3x) - x + C$

B54. $\int \sqrt{x}\,\ln x \, \mathrm{d}x$ Ans: $\frac{2}{9}x^{3/2}(3\ln x - 2) + C$

B55. $\int x^2 \ln x \, \mathrm{d}x$ Ans: $x^3(3\ln x - 1)/9 + C$

B56. $\int \tan^{-1} x \, \mathrm{d}x$ Ans: $x\tan^{-1} x - \frac{1}{2}\ln(1+x^2) + C$

B57. $\displaystyle\int \sin^{-1} x \, \mathrm{d}x$ Ans: $x \sin^{-1} x + \sqrt{1-x^2} + C$

Rational functions

B58. $\displaystyle\int \frac{2}{x^2-1} \, \mathrm{d}x$ Ans: $\ln|x-1| - \ln|x+1| + C$

B59. $\displaystyle\int \frac{5x+3}{x^2-9} \, \mathrm{d}x$ Ans: $2\ln|x+3| + 3\ln|x-3| + C$

B60. $\displaystyle\int \frac{x-11}{x^2+3x-4} \, \mathrm{d}x$ Ans: $3\ln|x+4| - 2\ln|x-1| + C$

B61. $\displaystyle\int \frac{x+1}{x^2-3x+2} \, \mathrm{d}x$ Ans: $3\ln|x-2| - 2\ln|x-1| + C$

B62. $\displaystyle\int \frac{x-1}{(x-4)^2} \, \mathrm{d}x$ Ans: $\ln|x-4| - 3/(x-4) + C$

B63. $\displaystyle\int \frac{x^2}{(x-1)(x-2)^2} \, \mathrm{d}x$ Ans: $\ln|x-1| - 4/(x-2) + C$

B64. $\displaystyle\int \frac{3x^2-8x+13}{(x+3)(x-1)^2} \, \mathrm{d}x$ Ans: $4\ln|x+3| - \ln|x-1| - 2/(x-1) + C$

B65. $\displaystyle\int \frac{2}{x(x^2+1)} \, \mathrm{d}x$ Ans: $\ln[x^2/(x^2+1)] + C$

B66. $\displaystyle\int \frac{x^3-4x}{(x^2+1)^2} \, \mathrm{d}x$ Ans: $\dfrac{1}{2}\ln(x^2+1) + \dfrac{5}{2(x^2+1)} + C$

Reduction formulae.

B67. Let $J_n = \displaystyle\int x^n \mathrm{e}^x \, \mathrm{d}x$. Show that $J_n = x^n \mathrm{e}^x - n\,J_{n-1}$ for $n = 1, 2, 3, \ldots$. Find J_0 and hence J_1 and J_2. Ans: $J_0 = \mathrm{e}^x + C$, $J_1 = (x-1)\mathrm{e}^x + C$, $J_2 = (x^2-2x+2)\mathrm{e}^x + C$

B68. Let $J_n = \displaystyle\int (\ln x)^n \, \mathrm{d}x$. Show that $J_n = x(\ln x)^n - nJ_{n-1}$ for $n = 1, 2, 3, \ldots$. Hence find J_2. Ans: $J_2 = x(\ln^2 x - 2\ln x + 2) + C$

B69. Let $J_n = \displaystyle\int \cos^n x \, \mathrm{d}x$. Show that $n\,J_n = \cos^{n-1} x \sin x + (n-1)J_{n-2}$ for $n = 2, 3, 4, \ldots$. Find J_0 and J_1, and hence deduce that

$$2J_2 = \cos x \sin x + x + C \quad \text{and} \quad 3J_3 = \cos^2 x \sin x + 2\sin x + C.$$

B70. Let $J_n = \displaystyle\int \tan^n x \, \mathrm{d}x$. Show that $J_n = (\tan^{n-1} x)/(n-1) - J_{n-2}$ for $n = 2, 3, 4, \ldots$. Deduce that $J_2 = \tan x - x + C$ and $J_3 = (\tan^2 x)/2 + \ln|\cos x| + C$.

B71. Let $J_n = \displaystyle\int \frac{\mathrm{d}x}{(x^2+1)^n}$. Show that $J_{n+1} = \dfrac{1}{2n}\left[(2n-1)J_n + \dfrac{x}{(x^2+1)^n}\right]$ for $n = 1, 2, 3, \ldots$. Deduce that $J_3 = \dfrac{x}{4(x^2+1)^2} + \dfrac{3x}{8(x^2+1)} + \dfrac{3}{8}\tan^{-1} x + C$.