ANALISI MATEMATICA I - A.A. 2011/2012

INTEGRALI INDEFINITI / ESERCIZI PROPOSTI

L'asterisco contrassegna gli esercizi più difficili.

1. Calcolare i seguenti integrali usando la linearità dell'integrale:

   a) $\int \frac{\left(x^2-3\right)\left(x^2+3\right)}{x^5} dx$ . . . . $\left[\log|x| + \frac{9}{4x^4} + c\right]$

   b) $\int \frac{x^2-4}{x-2} dx$ . . . . $\left[\frac{1}{2}x^2 + 2x + c\right]$

   c) $\int \frac{3x^3-3}{x-1} dx$ . . . . $\left[x^3 + \frac{3}{2}x^2 + 3x + c\right]$

   d) $\int \frac{x}{1+x} dx$ . . . . $[x - \log|x+1| + c]$

   e) $\int \frac{x^3+x^2+1}{x-1} dx$ . . . . $\left[\frac{1}{3}x^3 + x^2 + 2x + 3\log|x-1| + c\right]$

   f) $\int \frac{\cos^2 x}{1+\sin x} dx$ . . . . $[x + \cos x + c]$

   g) $\int \tan^2 x\, dx$ . . . . $[\tan x - x + c]$

2. Calcolare i seguenti integrali immediati usando la regola di integrazione per sostituzione:

   a) $\int e^{\sin x} \cos x\, dx$ . . . . $\left[e^{\sin x} + c\right]$

   b) $\int \cos^5 x \sin x\, dx$ . . . . $\left[-\frac{\cos^6 x}{6} + c\right]$

   c) $\int \frac{\cos(\log x)}{x} dx$ . . . . $[\sin(\log x) + c]$

   d) $\int \frac{1}{x\sqrt{\log x}} dx$ . . . . $\left[2\sqrt{\log x} + c\right]$

   e) $\int \frac{1}{\sqrt{3x+1}} dx$ . . . . $\left[\frac{2}{3}\sqrt{3x+1} + c\right]$

   f) $\int \frac{1}{\cos^2(5x+9)} dx$ . . . . $\left[\frac{1}{5}\tan(5x+9) + c\right]$

   g) $\int x^3 \sin\left(x^4\right) dx$ . . . . $\left[-\frac{\cos x^4}{4} + c\right]$

   h) $\int x e^{2x^2-1} dx$ . . . . $\left[\frac{1}{4}e^{2x^2-1} + c\right]$

   i) $\int x\sqrt{1+x^2} dx$ . . . . $\left[\frac{1}{3}\left(1+x^2\right)\sqrt{1+x^2} + c\right]$

   l) $\int \frac{x^2}{\sqrt{1-x^3}} dx$ . . . . $\left[-\frac{2}{3}\sqrt{1-x^3} + c\right]$

   m) $\int \frac{x}{1+x^4} dx$ . . . . $\left[\frac{1}{2}\arctan x^2 + c\right]$

3. Calcolare i seguenti integrali usando la regola di integrazione per parti:
   a) $\int x^2 \log x \, dx$ .... $\left[\frac{1}{3}x^3\left(\log x - \frac{1}{3}\right) + c\right]$
   b) $\int x^2 \cos x \, dx$ .... $\left[\left(x^2 - 2\right)\sin x + 2x\cos x + c\right]$
   c) $\int x^2 e^{2x} dx$ .... $\left[\left(x^2 - x + \frac{1}{2}\right)\frac{e^{2x}}{2} + c\right]$
   d) $\int x \cosh(3x) \, dx$ .... $\left[\frac{1}{3}x\sinh 3x - \frac{1}{9}\cosh 3x + c\right]$
   e) $\int (x+5)\log x \, dx$ .... $\left[\left(\frac{x^2}{2} + 5x\right)\log x - \frac{x^2}{4} - 5x + c\right]$
   f) $\int \frac{\log x}{x^2} dx$ .... $\left[-\frac{1+\log x}{x} + c\right]$
   g) $\int \arctan x \, dx$ .... $\left[x\arctan x - \frac{1}{2}\log\left(1 + x^2\right) + c\right]$
   h) $\int \arcsin x \, dx$ .... $\left[x\arcsin x + \sqrt{1 - x^2} + c\right]$
   i) $\int x\cos^2 x \, dx$ .... $\left[\frac{1}{2}\left(\frac{x\sin 2x}{2} + \frac{\cos 2x}{4} + \frac{x^2}{2}\right) + c\right]$
   l) $\int e^{2x}\cos x \, dx$ .... $\left[\frac{\sin x + 2\cos x}{5}e^{2x} + c\right]$
4. Calcolare i seguenti integrali di funzioni razionali:
   a) $\int \frac{x+2}{x^2 - 4x + 4} dx$ .... $\left[\log|x-2| - \frac{4}{x-2} + c\right]$
   b) $\int \frac{2x+5}{x^2 + 2x - 3} dx$ .... $\left[\frac{7}{4}\log|x-1| + \frac{1}{4}\log|x+3| + c\right]$
   c) $\int \frac{x+1}{x^2 - 2x + 4} dx$ .... $\left[\frac{1}{2}\log(x^2 - 2x + 4) + \frac{2}{\sqrt{3}}\arctan\frac{x-1}{\sqrt{3}} + c\right]$
   d) $\int \frac{x+1}{x^2 - x + 5} dx$ .... $\left[\frac{1}{2}\log(x^2 - x + 5) + \frac{3}{\sqrt{19}}\arctan\frac{2x-1}{\sqrt{19}} + c\right]$
   e) $\int \frac{2x^2 + x}{(x+2)(x^2 + 2x + 6)} dx$ .... $\left[\log|x+2| + \frac{1}{2}\log\left(x^2 + 2x + 6\right) - \frac{4}{\sqrt{5}}\arctan\frac{x+1}{\sqrt{5}} + c\right]$
   f) $\int \frac{3x-1}{(x-1)(x-2)^2} dx$ .... $\left[2\log\left|\frac{x-1}{x-2}\right| - \frac{5}{x-2} + c\right]$
   g) $\int \frac{x^4 - 5x^3 + 8x^2 - 9x + 11}{x^2 - 5x + 6} dx$ .... $\left[\frac{x^3}{3} + 2x - \log|x-2| + 2\log|x-3| + c\right]$
   h) $\int \frac{1}{(x^2+1)^2} dx$ .... $\left[\frac{1}{2}\frac{x}{1+x^2} + \frac{1}{2}\arctan x + c\right]$
   i) $\int \frac{1}{(x^2+1)^3} dx$ .... $\left[\frac{1}{8}\frac{3x^3 + 5x}{(x^2+1)^2} + \frac{3}{8}\arctan x + c\right]$
   l) $\int \frac{x^2}{(x+1)^4} dx$ .... $\left[-\frac{1}{3}\frac{3x^2 + 3x + 1}{(x+1)^3} + c\right]$
5. Calcolare i seguenti integrali mediante opportune sostituzioni:
   a) $\int \frac{e^x + 1}{e^{2x} + 1} dx$ .... $\left[x - \frac{1}{2}\log\left(e^{2x} + 1\right) + \arctan e^x + c\right]$
   b) $\int \frac{(5e^x + 4)e^x}{(e^x - 2)(e^{2x} + e^x + 1)} dx$ .... $\left[2\log(e^x - 2) - \log\left(e^{2x} + e^x + 1\right) + c\right]$

c) $\int x\sqrt{1-x}dx$ .... $\left[\frac{2}{5}(1-x)^2\sqrt{1-x}-\frac{2}{3}(1-x)\sqrt{1-x}+c\right]$

d) $\int \frac{1+\sqrt{x}}{x(1+\sqrt[3]{x})}dx$ .... $[6\sqrt[6]{x}+\log x-\log(1+\sqrt[3]{x})-\arctan\sqrt[6]{x}+c]$

e) $\int \frac{1}{\cos x}dx$ .... $\left[\log\left|\frac{1+\sin x}{\cos x}\right|+c\right]$

f) $\int \frac{1}{2\sin x+\cos x-1}dx$ .... $\left[\frac{1}{2}\log\left|\frac{\tan(x/2)}{\tan(x/2)-2}\right|+c\right]$

g) $\int \frac{\cos^2 x}{1-2\sin^2 x}dx$ .... $\left[\frac{1}{4}\log\left|\frac{\sin x+\cos x}{\sin x-\cos x}\right|+\frac{1}{2}x+c\right]$

h) $\int \tan^3 x\,dx$ .... $\left[\frac{1}{2}\tan^2 x-\frac{1}{2}\log\left(1+\tan^2 x\right)+c\right]$

6. Calcolare i seguenti integrali mediante la sostituzione suggerita a fianco:

a) $\int \sqrt{x^2-1}\,dx, \quad x=\cosh t$ .... $\left[\frac{x}{2}\sqrt{x^2-1}-\frac{1}{2}\cosh^{-1}x+c\right]$

b) $\int \sqrt{x^2+1}\,dx, \quad x=\sinh t$ .... $\left[\frac{x}{2}\sqrt{(1+x^2)}+\frac{1}{2}\sinh^{-1}x+c\right]$

c*) $\int \sqrt{\frac{1-x}{1+x}}\,dx, \quad x=\sin t$ .... $\left[\arcsin x+\sqrt{1-x^2}+c\right]$

d) $\int \frac{1}{(x^2+1)\sqrt{x^2+1}}dx, \quad x=\sinh t$ .... $\left[\frac{1}{\sqrt{x^2+1}}+c\right]$

e) $\int \frac{\sqrt{x^2+1}}{x^2}dx, \quad x=\sinh t$ .... $\left[\sinh^{-1}x-\frac{\sqrt{x^2+1}}{x}+c\right]$

f) $\int \frac{\sqrt{x^2-1}}{x}dx, \quad t=\sqrt{x^2-1}$ .... $\left[\sqrt{x^2-1}-\arctan\left(\sqrt{x^2-1}\right)+c\right]$

g*) $\int x\sqrt{x^2+x+1}\,dx, \quad x=\frac{1}{2}\left(\sqrt{3}\sinh t-1\right)$

.... $\left[\frac{1}{3}\left(x^2+x+1\right)^{3/2}-\frac{1}{8}(2x+1)\sqrt{x^2+x+1}-\frac{3}{16}\sinh^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)+c\right]$

7. Per ciascuno degli integrali a) degli esercizi 1-6 precedenti, determinare la primitiva $F(x)$ della funzione integranda che si annulla nel punto $x_0=1$.

8. Calcolare i seguenti integrali:

a) $\int \frac{x}{\sqrt{1-x^4}}dx$ .... $\left[\frac{1}{2}\arcsin\left(x^2\right)+c\right]$

b) $\int \frac{x}{\sqrt{x-1}}dx$ .... $\left[2\sqrt{x-1}+\frac{2}{3}(x-1)\sqrt{x-1}+c\right]$

c) $\int \frac{1}{\sqrt{x}\,(\sqrt[4]{x}-1)}dx$ .... $[4\sqrt[4]{x}+4\log|\sqrt[4]{x}-1|+c]$

d) $\int \frac{1}{\sqrt{1-4x^2}}dx$ .... $\left[\frac{1}{2}\arcsin 2x+c\right]$

e*) $\int \sqrt{9x^2-1}dx$ .... $\left[\frac{x}{2}\sqrt{9x^2-1}-\frac{1}{6}\cosh^{-1}(3x)+c\right]$

f) $\int \frac{1}{e^x+1}dx$ .... $[x-\log(e^x+1)+c]$

g) $\int e^{x+e^x}dx$ .... $\left[e^{e^x}+c\right]$

h*) $\int \sqrt{e^x-1}dx$ .... $\left[2\sqrt{e^x-1}-2\arctan\left(\sqrt{e^x-1}\right)+c\right]$

i*) $\displaystyle\int \frac{1}{\sqrt{e^{-2x}-1}}dx$ .... $[\arcsin(e^x)+c]$

l) $\displaystyle\int \frac{1}{x+x\log x}dx$ .... $[\log|1+\log x|+c]$

m) $\displaystyle\int \frac{\log x}{x\sqrt{4+3\log^2 x}}dx$ .... $\left[\frac{1}{3}\sqrt{4+3\log^2 x}+c\right]$

n) $\displaystyle\int \frac{\log x}{\sqrt{x}}dx$ .... $[2\sqrt{x}(\log x-2)+c]$

o) $\displaystyle\int \sin^2 x\,dx$ .... $\left[\frac{x-\sin x\cos x}{2}+c\right]$

p) $\displaystyle\int \sin^3 x\,dx$ .... $\left[\frac{\cos^3 x}{3}-\cos x+c\right]$

q) $\displaystyle\int \frac{1}{\sin x\cos x}dx$ .... $[\log|\tan x|+c]$

r) $\displaystyle\int \frac{1}{\sin^2 x\cos^2 x}dx$ .... $[\tan x-\cot x+c]$

s) $\displaystyle\int \frac{\cos x}{3-\cos^2 x}dx$ .... $\left[\frac{1}{\sqrt{2}}\arctan\left(\frac{\sin x}{\sqrt{2}}\right)+c\right]$

t) $\displaystyle\int \frac{4\sin x}{4\cos^2 x-8\cos x+5}dx$ .... $[-2\arctan(2\cos x-2)+c]$

u*) $\displaystyle\int x\arctan(1+16x)\,dx$ .... $\left[\frac{x^2}{2}\arctan(1+16x)+\frac{1}{512}\log\left(1+(1+16x)^2\right)-\frac{x}{32}+c\right]$

9. Per ciascuna delle seguenti funzioni definite a tratti (continue sul proprio dominio), calcolare tutte le primitive $F(x)$ e determinare quella che vale 1 in $x_0=0$:

a) $f(x)=\begin{cases} \dfrac{x+2}{x-1} & \text{se } x<0 \\ xe^{3x}-2 & \text{se } x\geq 0 \end{cases}$ ..... $\left[F(x)=\begin{cases} x+3\log|x-1|+c & \text{se } x<0 \\ \left(x-\frac{1}{3}\right)\frac{e^{3x}}{3}-2x+c+\frac{1}{9} & \text{se } x\geq 0 \end{cases},\ c=1\right]$

b) $f(x)=\begin{cases} -x^3\sin\left(\pi+\pi x^2\right) & \text{se } x\leq 1 \\ x^2-8x+7 & \text{se } x>1 \end{cases}$

.... $\left[F(x)=\begin{cases} \frac{\sin\left(\pi x^2\right)}{2\pi^2}-x^2\frac{\cos\left(\pi x^2\right)}{2\pi}+c & \text{se } x\leq 1 \\ \frac{x^3}{3}-4x^2+7x+c+\frac{1}{2\pi}-\frac{10}{3} & \text{se } x>1 \end{cases},\ c=1\right]$

c) $f(x)=\begin{cases} \log\left(1+25x^2\right) & \text{se } x\leq 1 \\ x\log 26 & \text{se } x>1 \end{cases}$

.... $\left[F(x)=\begin{cases} x\log\left(1+25x^2\right)-2x+\frac{2}{5}\arctan 5x+c & \text{se } x\leq 1 \\ \frac{\log 26}{2}x^2+c & \text{se } x>1 \end{cases},\ c=1\right]$

d) $f(x)=\begin{cases} 4x+1 & \text{se } x\leq 0 \\ \dfrac{1}{\sqrt{4-x}-3} & \text{se } 0<x\leq 4 \end{cases}$

.... $\left[F(x)=\begin{cases} 2x^2+x+c-4 & \text{se } x<0 \\ -2\sqrt{4-x}-6\log\left(3-\sqrt{4-x}\right)+c & \text{se } 0<x\leq 4 \end{cases},\ c=5\right]$

ALTRE SOLUZIONI.

Esercizio 7.

a1) $F(x) = \log|x| + \frac{9}{4x^4} - \frac{9}{4}$

a2) $F(x) = e^{\sin x} - e^{\sin 1}$

a3) $F(x) = \frac{1}{3}x^3\left(\log x - \frac{1}{3}\right) + \frac{1}{9}$

a4) $F(x) = \log|x-2| - \frac{4}{x-2} - 4$

a5) $F(x) = x - \frac{1}{2}\log\left(e^{2x}+1\right) + \arctan e^x - 1 + \frac{1}{2}\log\left(e^2+1\right) - \arctan e$

a6) $F(x) = \frac{x}{2}\sqrt{x^2-1} - \frac{1}{2}\cosh^{-1} x$