

Integrali semplici

$$(a) \int \frac{1 + \cos x}{x + \sin x} dx \quad [\log |x + \sin x| + c, \quad c \in \mathbb{R}]$$

$$(b) \int \frac{3x + 2}{x^2 + 1} dx \quad \left[\frac{3}{2} \log(x^2 + 1) + 2 \arctan x + c, \quad c \in \mathbb{R} \right]$$

$$(c) \int \frac{dx}{\sin^2 x \cos^2 x}. \quad [\tan x - \cot x + c, \quad c \in \mathbb{R}]$$

Integrazione per parti

$$(a) \int \arcsin x dx \quad [x \arcsin x + \sqrt{1 - x^2} + c, \quad c \in \mathbb{R}]$$

$$(b) \int x^2 \log^2 x dx \quad \left[\frac{1}{3} x^3 \left(\log^2 x - \frac{2}{3} \log x + \frac{2}{9} \right) + c, \quad c \in \mathbb{R} \right]$$

$$(c) \int x^3 \sqrt{2 - x^2} dx. \quad \left[-\frac{1}{3} x^2 (2 - x^2)^{\frac{3}{2}} - \frac{2}{15} (2 - x^2)^{\frac{5}{2}} + c, \quad c \in \mathbb{R} \right]$$

Integrazione per sostituzione

$$(a) \int \frac{dx}{x \log^3 x} \quad \left[-\frac{1}{2 \log^2 x} + c, \quad c \in \mathbb{R} \right]$$

$$(b) \int \frac{\sin 2x}{1 + \sin^2 x} dx \quad [\log(1 + \sin^2 x) + c, \quad c \in \mathbb{R}]$$

$$(c) \int \frac{dx}{\sqrt{x} + \sqrt[3]{x}}. \quad [2\sqrt{x} - 3\sqrt[3]{x} + 6\sqrt[6]{x} - 6 \log(\sqrt[6]{x} + 1) + c, \quad c \in \mathbb{R}]$$

Integrazione delle funzioni razionali fratte

$$(a) \int \frac{x + 1}{x(1 + x^2)} dx \quad \left[\log \frac{|x|}{\sqrt{1 + x^2}} + \arctan x + c, \quad c \in \mathbb{R} \right]$$

$$(b) \int \frac{1}{x^3(1 + x^2)} dx \quad \left[\log \frac{\sqrt{1 + x^2}}{|x|} - \frac{1}{2x^2} + c, \quad c \in \mathbb{R} \right]$$

$$(c) \int \frac{x^3 + x^2 - x}{x^2 + x - 6} dx \quad \left[\frac{1}{2} x^2 + 2 \log |x - 2| + 3 \log |x + 3| + c, \quad c \in \mathbb{R} \right]$$

$$(d) \int \frac{dx}{x(x^2 + 2x + 3)} \quad \left[\log \sqrt[6]{\frac{x^2}{x^2 + 2x + 3}} - \frac{\sqrt{2}}{6} \arctan \frac{x + 1}{\sqrt{2}} + c, \quad c \in \mathbb{R} \right]$$

$$(e) \int \frac{x^2 - 10x + 10}{x^3 + 2x^2 + 5x} dx. \quad \left[\log \frac{x^2}{\sqrt{x^2 + 2x + 5}} - \frac{13}{2} \arctan \frac{x + 1}{2} + c, \quad c \in \mathbb{R} \right]$$

Integrazione con sostituzioni speciali

- (a) $\int \frac{1}{\sin x} dx$ $\left[\log \left| \tan \frac{x}{2} \right| + c, \quad c \in \mathbb{R} \right]$
- (b) $\int \frac{dx}{x^2 \sqrt{4+x^2}}$ $\left[-\frac{1}{x^2 + x\sqrt{x^2+4}} + c, \quad c \in \mathbb{R} \right]$
- (c) $\int \frac{dx}{\sqrt{1+2x-x^2}}$ $\left[\arcsin \frac{x-1}{\sqrt{2}} + c, \quad c \in \mathbb{R} \right]$
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Integrazione di funzioni definite a tratti

- (a) $f(x) = \begin{cases} xe^x & \text{se } x \leq 0 \\ \sin x & \text{se } x > 0 \end{cases}$ $\left[\begin{cases} e^x(x-1) + c & \text{se } x \leq 0 \\ -\cos x + c & \text{se } x > 0, \end{cases} \quad c \in \mathbb{R} \right]$
- (b) $f(x) = \begin{cases} -x^3 \sin(\pi + \pi x^2) & \text{se } x \leq 1 \\ x^2 - 8x + 7 & \text{se } x > 1. \end{cases}$
- $\left[\begin{cases} -\frac{1}{2\pi} x^2 \cos(\pi x^2) + \frac{1}{2\pi^2} \sin(\pi x^2) + c & \text{se } x \leq 1 \\ \frac{1}{3} x^3 - 4x^2 + 7x + c + \frac{1}{2\pi} - \frac{10}{3} & \text{se } x > 1, \end{cases} \quad c \in \mathbb{R} \right]$
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Integrali definiti

- (a) $\int_0^\pi |6x - \pi| \sin x dx$ $[6\pi - 6]$
- (b) $\int_{-\frac{\pi}{2}}^0 \frac{2 \sin^2 x + 3 \sin x + 3}{(\sin x - 1)(\sin^2 x + 3)} \cos x dx$ $\left[\frac{\sqrt{3}}{6} \pi - 2 \log 2 \right]$
- (c) $\int_e^{e^{\frac{3}{2}}} \frac{1}{x(1 - \sqrt{\log x - 1})} dx.$ $\left[-\sqrt{2} - 2 \log \left(1 - \frac{\sqrt{2}}{2} \right) \right]$
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Altri esercizi

Calcolare l'area delle seguenti regioni di piano:

- (a) $A = \left\{ (x, y) \in \mathbb{R}^2 : 1 \leq x \leq 2, 0 \leq y \leq \frac{1}{x(1 - \log^2 x)} \right\}$ $\left[\frac{1}{2} \log \left(\frac{1 + \log 2}{1 - \log 2} \right) \right]$
- (b) $B = \left\{ (x, y) \in \mathbb{R}^2 : -\sqrt{5} \leq x \leq -1, \frac{x}{x^2 + 2\sqrt{x^2 - 1}} \leq y \leq 0 \right\}$ $\left[\log 3 - \frac{2}{3} \right]$
- (c) $C = \left\{ (x, y) \in \mathbb{R}^2 : 1 \leq x \leq e, \frac{\log x}{x\sqrt{4 + 3 \log^2 x}} \leq y \leq x^2 \right\}.$ $\left[\frac{1}{3} (e^3 + 1 - \sqrt{7}) \right]$