



Calcolare i seguenti integrali indefiniti

Esercizio 1

$$\int \left(4\sqrt[3]{x} - \frac{x}{2} + \frac{1}{x} \right) dx$$



$$\int \left(4\sqrt[3]{x} - \frac{x}{2} + \frac{1}{x} \right) dx = 4 \int x^{1/3} dx - \frac{1}{2} \int x dx + \int \frac{dx}{x} =$$

$$= 4 \cdot \frac{x^{\frac{1}{3}+1}}{\frac{1}{3}+1} - \frac{1}{2} \cdot \frac{x^{1+1}}{(1+1)} + \ln|x| + C =$$

$$= 3x^{4/3} - \frac{x^2}{4} + \ln|x| + C =$$

$$= 3x\sqrt[3]{x} - \frac{x^2}{4} + \ln|x| + C$$

Esercizio 2

$$\int \frac{5x^2 + 7x - 3\sqrt{x}}{4x^2} dx$$



$$\int \frac{5x^2 + 7x - 3\sqrt{x}}{4x^2} dx = \frac{5}{4} \int dx + \frac{7}{4} \int \frac{dx}{x} - \frac{3}{4} \int \frac{x^{1/2}}{x^2} dx = \frac{5}{4}x + \frac{7}{4} \ln|x| - \frac{3}{4} \int x^{-3/2} dx$$

$$= \frac{5}{4}x + \frac{7}{4} \ln|x| - \frac{3}{4} \cdot \frac{x^{-3/2+1}}{-3/2+1} + C = \frac{5}{4}x + \frac{7}{4} \ln|x| + \frac{3}{4} \frac{x^{-1/2}}{\left(-\frac{1}{2}\right)} + C =$$

$$= \frac{5}{4}x + \frac{7}{4} \ln|x| - \frac{3}{2\sqrt{x}} + C$$

**Esercizio 3**

$$\int \left(\frac{x^2}{3} + 3x - \sqrt[3]{x} \right) dx$$



$$\int \left(\frac{x^2}{3} + 3x - \sqrt[3]{x} \right) dx = \frac{1}{3} \int x^2 dx + 3 \int x dx - \int x^{1/3} dx =$$

$$= \frac{1}{3} \cdot \frac{x^{2+1}}{(2+1)} + 3 \cdot \frac{x^{1+1}}{(1+1)} - \frac{x^{1+1/3}}{\left(1+\frac{1}{3}\right)} + C = \frac{x^3}{9} + \frac{3}{2}x^2 - \frac{3}{2}x^{4/3} + C =$$

$$= \frac{x^3}{9} + \frac{3}{2}x^2 - \frac{3}{2}\sqrt[3]{x^4} + C = \frac{x^3}{9} + \frac{3}{2}x^2 - \frac{3}{2}\sqrt[3]{x^3} \cdot \sqrt[3]{x} + C =$$

$$\frac{x^3}{9} + \frac{3}{2}x^2 - \frac{3}{2}x\sqrt[3]{x} + C$$

Esercizio 4

$$\int \frac{x^4 + 2}{3x^4} dx$$



$$\int \frac{x^4 + 2}{3x^4} dx = \int \frac{x^4}{3x^4} dx + \int \frac{2}{3x^4} dx = \frac{1}{3} \int dx + \frac{2}{3} \int \frac{dx}{x^4} = \frac{1}{3} \int dx + \frac{2}{3} \int x^{-4} dx =$$

$$= \frac{x}{3} + \frac{2}{3} \cdot \frac{x^{-4+1}}{(-4+1)} + C = \frac{x}{3} + \frac{2}{3} \cdot \frac{1}{3} \cdot x^{-3} + C =$$

$$= \frac{x}{3} + \frac{2}{9x^3} + C$$

**Esercizio 5**

$$\int \sin^3 x \cdot \cos x \, dx$$



In base alla
$$\int f^n(x) \cdot f'(x) \, dx = \frac{f^{n+1}(x)}{n+1} + C$$

considerando $f(x) = \sin x$ e $f'(x) = \cos x$

$$\int \sin^3 x \cdot \cos x \, dx = \frac{\sin^{(3+1)} x}{3+1} + C = \frac{1}{4} \sin^4 x + C$$

Esercizio 6

$$\int \frac{3x^2}{1+x^3} \, dx$$



Si nota che posto $f(x) = 1+x^3$ è $f'(x) = 3x^2$. In base alla
$$\int \frac{f'(x)}{f(x)} \, dx = \ln |f(x)| + C$$

$$\int \frac{3x^2}{1+x^3} \, dx = \ln |1+x^3| + C$$

Esercizio 7

$$\int x\sqrt{x^2+3} \, dx$$



se $f(x) = 3+x^2$ è $f'(x) = 2x$. moltiplicando per $\frac{2}{2}$
$$\int x\sqrt{x^2+3} \, dx = \frac{1}{2} \int 2x\sqrt{x^2+3} \, dx$$

In base alla
$$\int f^n(x) \cdot f'(x) \, dx = \frac{f^{n+1}(x)}{n+1} + C$$

$$\frac{1}{2} \int 2x\sqrt{x^2+3} \, dx = \frac{1}{2} \cdot \frac{(x^2+3)^{1+1/2}}{\left(\frac{1}{2}+1\right)} + C = \frac{1}{2} \cdot \frac{2}{3} \cdot (x^2+3)^{3/2} + C =$$

$$= \frac{1}{3} \cdot \sqrt{(x^2+3)^3} + C = \frac{(x^2+3)}{3} \sqrt{(x^2+3)} + C$$

**Esercizio 8**

$$\int \frac{\sin x}{\sqrt[3]{2+3\cos x}} dx$$



$$\int \frac{\sin x}{\sqrt[3]{2+3\cos x}} dx = -\frac{1}{3} \int \frac{-3\sin x}{\sqrt[3]{2+3\cos x}} dx = -\frac{1}{3} \int (-3\sin x)(2+3\cos x)^{-1/3} dx$$

se $f(x)=2+3\cos x$ si ha $f'(x)=-3\sin x$ si applica la $\int f^n(x) \cdot f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$

$$\int \frac{\sin x}{\sqrt[3]{2+3\cos x}} dx = -\frac{1}{3} \int (-3\sin x)(2+3\cos x)^{-1/3} dx = \frac{(2+3\cos x)^{1-1/3}}{\left(-\frac{1}{3}+1\right)} + C$$

$$= -\frac{1}{3} \cdot \frac{3}{2} \cdot (2+3\cos x)^{2/3} + C = -\frac{1}{2} \cdot \sqrt[3]{(2+3\cos x)^2} + C$$

Esercizio 9

$$\int \frac{1}{x \ln x} dx$$



$$\int \frac{1}{x \ln x} dx = \int \frac{1/x}{\ln x} dx$$

se $f(x)=\ln x$ si ha $f'(x)=1/x$ siamo nel caso $\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C$ per cui

$$\int \frac{1}{x \ln x} dx = \int \frac{1/x}{\ln x} dx = \ln |\ln x| + C$$

Esercizio 10

$$\int \operatorname{tg} x dx$$



$$\int \operatorname{tg} x dx = \int \frac{\sin x}{\cos x} dx$$

ricordando che $D[\cos x] = -\sin x$ si ha $f(x)=\cos x$ e $f'(x)=-\sin x$ applichiamo la:

$$\int \frac{f'(x)}{f(x)} dx = \ln |f(x)| + C \quad \text{ottenendo:} \quad \int \operatorname{tg} x dx = -\int \frac{-\sin x}{\cos x} dx = -\ln |\cos x| + C$$

**Esercizio 11**

$$\int \frac{x+2}{x+3} dx$$



$$\int \frac{x+2}{x+3} dx = \int \frac{x+2+1-1}{x+3} dx = \int \frac{x+3-1}{x+3} dx = \int \frac{x+3}{x+3} dx - \int \frac{1}{x+3} dx =$$

$$\int dx - \int \frac{1}{x+3} dx$$

nel secondo integrale si nota che $D[x+3]=1$ siamo nel caso $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$

$$\int dx - \int \frac{1}{x+3} dx = x + \ln|x+3| + C$$

Esercizio 12

$$\int \frac{4x^3+5}{2x+3} dx$$



dopo aver eseguito la divisione fra i due polinomi

$$\int \frac{4x^3+5}{2x+3} dx = \int \left(2x^2 - 3x + \frac{9}{2} \right) dx - \int \frac{17/2}{2x+3} dx = \int 2x^2 dx - 3 \int x dx + \frac{9}{2} \int dx - \frac{17}{2} \int \frac{1}{2x+3} dx$$

$$= \int 2x^2 dx - 3 \int x dx + \frac{9}{2} \int dx - \frac{17}{4} \int \frac{2}{2x+3} dx =$$

$$= \frac{2}{3} x^3 - \frac{3}{2} x^2 + \frac{9}{2} x - \frac{17}{4} \ln|2x+3| + C$$

infatti l'ultimo integrale è riconducibile alla forma $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$

**Esercizio 13**

$$\int \frac{dx}{x^2 - 3x + 2}$$



dopo aver eseguito la soluzione del trinomio di II grado

$$\int \frac{dx}{(x-1)(x-2)}$$

poniamo

$$\frac{1}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2} = \frac{Ax - 2A + Bx - B}{(x-1)(x-2)} \quad \text{deve essere soddisfatto il sistema}$$

$$\begin{cases} A + B = 0 \\ -2A - B = 1 \end{cases} \quad \begin{cases} B = -A \\ -2A + A = 1 \end{cases} \longrightarrow A = -1 \longrightarrow B = 1$$

$$\int \frac{dx}{(x-1)(x-2)} = -\int \frac{dx}{x-1} + \int \frac{dx}{x-2} = -\ln|x-1| + \ln|x-2| + C = \ln\left|\frac{x-2}{x-1}\right| + C$$

Esercizio 14

$$\int \frac{dx}{x^2 - 4}$$



$$\int \frac{dx}{x^2 - 4} = \int \frac{dx}{(x-2)(x+2)} \quad \text{ponendo}$$

$$\frac{1}{(x-2)(x+2)} = \frac{A}{x-2} + \frac{B}{x+2} = \frac{Ax + 2A + Bx - 2B}{(x-2)(x+2)}$$

deve essere soddisfatto il sistema

$$\begin{cases} A + B = 0 \\ 2A - 2B = 1 \end{cases} \quad \begin{cases} B = -A \\ 2A + 2A = 1 \end{cases} \longrightarrow A = \frac{1}{4} \longrightarrow B = -\frac{1}{4}$$

$$\int \frac{dx}{x^2 - 4} = \int \frac{dx}{(x-2)(x+2)} = \frac{1}{4} \int \frac{dx}{x-2} - \frac{1}{4} \int \frac{dx}{x+2} =$$

$$= \frac{1}{4} \ln|x-2| - \frac{1}{4} \ln|x+2| + C = \frac{1}{4} \ln\left|\frac{x-2}{x+2}\right| + C$$

**Esercizio 15**

$$\int \frac{1+2x}{2x^2+x-3} dx$$



risolvendo il denominatore con la formula del trinomio di II° grado:

$$2x^2+x-3=2\left(x+\frac{3}{2}\right)(x-1)=(2x+3)(x-1) \text{ poniamo}$$

$$\begin{aligned} \frac{2x+1}{2x^2+x-3} &= \frac{2x+1}{(2x+3)(x-1)} = \frac{A}{(2x+3)} + \frac{B}{(x-1)} = \frac{Ax-A+2Bx+3B}{(2x+3)(x-1)} = \\ &= \frac{(A+2B)x+(3B-A)}{(2x+3)(x-1)} \end{aligned}$$

deve essere soddisfatto il sistema

$$\begin{cases} A+2B=2 \\ 3B-A=1 \end{cases} \begin{cases} A=2-2B \\ 3B-2+2B=1 \end{cases} \longrightarrow B=\frac{3}{5} \longrightarrow A=2-2\cdot\frac{3}{5}=\frac{4}{5}$$

$$\int \frac{1+2x}{2x^2+x-3} dx = \frac{4}{5} \int \frac{dx}{2x+3} + \frac{3}{5} \int \frac{dx}{x-1} = \frac{2}{5} \int \frac{2}{2x+3} dx + \frac{3}{5} \int \frac{dx}{x-1} =$$

$$= \frac{2}{5} \ln|2x+3| + \frac{3}{5} \ln|x-1| + C$$

**Esercizio 16**

$$\int \frac{dx}{9x^2 - 6x + 1}$$



Si riconosce come sia $9x^2 - 6x + 1 = 9 \cdot \left(x - \frac{1}{3}\right)^2$

$$\int \frac{dx}{9x^2 - 6x + 1} = \frac{1}{9} \int \left(x - \frac{1}{3}\right)^{-2} dx$$

facendo riferimento alla $\int f^n(x) \cdot f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$

ponendo $f(x) = x - \frac{1}{3}$ ed $f'(x) = 1$

$$\begin{aligned} \int \frac{dx}{9x^2 - 6x + 1} &= \frac{1}{9} \int \left(x - \frac{1}{3}\right)^{-2} dx = \frac{1}{9} \cdot \frac{\left(x - \frac{1}{3}\right)^{-2+1}}{(-2+1)} + C = -\frac{1}{9 \cdot \left(x - \frac{1}{3}\right)} + C = \\ &= -\frac{1}{9x-3} + C = -\frac{1}{3(3x-1)} + C \end{aligned}$$

Esercizio 17

$$\int \frac{dx}{4x^2 + 12x + 9}$$



$$\int \frac{dx}{4x^2 + 12x + 9} = \int \frac{dx}{(2x+3)^2} = \frac{1}{2} \int \frac{2}{(2x+3)^2} dx$$

usando la $\int f^n(x) \cdot f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$

ponendo $f(x) = 2x+3$ ed $f'(x) = 2$

$$\int \frac{dx}{4x^2 + 12x + 9} = \int \frac{dx}{(2x+3)^2} = \frac{1}{2} \cdot \frac{(2x+3)^{-2+1}}{(-2+1)} + C = -\frac{1}{2(2x+3)} + C$$

**Esercizio 18**

$$\int \frac{x+5}{x^2-6x+9} dx$$



$$\int \frac{x+5}{x^2-6x+9} dx = \frac{1}{2} \int \frac{2x+10}{x^2-6x+9} dx = \frac{1}{2} \int \frac{2x+10-6+6}{x^2-6x+9} dx =$$

$$= \frac{1}{2} \int \frac{(2x-6)+16}{x^2-6x+9} dx = \frac{1}{2} \int \frac{2x-6}{x^2-6x+9} dx + \frac{1}{2} \int \frac{16}{x^2-6x+9} dx =$$

$$\frac{1}{2} \int \frac{2x-6}{x^2-6x+9} dx + 8 \int \frac{dx}{x^2-6x+9}$$

il primo dei due è facilmente risolvibile usando la regola $\int \frac{f'(x)}{f(x)} = \ln|f(x)| + C$

$$\frac{1}{2} \int \frac{2x-6}{x^2-6x+9} dx = \frac{1}{2} \ln|x^2-6x+9| + C = \frac{1}{2} \ln|x-3|^2 + C = \ln|x-3| + C$$

per il secondo si osserva che $x^2-6x+9 = (x-3)^2$ ponendo $f(x)=x-3$ ed $f'(x)=1$

si può applicare la $\int f^n(x) \cdot f'(x) dx = \frac{f^{n+1}(x)}{n+1} + C$

$$8 \int \frac{dx}{x^2-6x+9} = 8 \int \frac{dx}{(x-3)^2} = 8 \cdot \frac{(x-3)^{-2+1}}{(-2+1)} + C = -\frac{8}{(x-3)} + C$$

$$\int \frac{x+5}{x^2-6x+9} dx = \ln|x-3| - \frac{8}{(x-3)} + C$$

**Esercizio 19**

$$\int \frac{dx}{x^3 - 6x^2 + 11x - 6}$$



$$\int \frac{dx}{x^3 - 6x^2 + 11x - 6} = \int \frac{dx}{(x-1)(x-2)(x-3)}$$

poniamo

$$\frac{1}{(x-1)(x-2)(x-3)} = \frac{A}{x-1} + \frac{B}{x-2} + \frac{C}{x-3} \quad \text{ottenendo}$$

$$1 = A(x^2 - 5x + 6) + B(x^2 - 4x + 3) + C(x^2 - 3x + 2)$$

$$1 = (A + B + C)x^2 + (-5A - 4B - 3C)x + (6A + 3B + 2C)$$

$$\begin{cases} A + B + C = 0 \\ -5A - 4B - 3C = 0 \\ 6A + 3B + 2C = 1 \end{cases} \longrightarrow \begin{cases} A = \frac{1}{2} \\ B = -1 \\ C = \frac{1}{2} \end{cases}$$

$$\int \frac{dx}{x^3 - 6x^2 + 11x - 6} = \frac{1}{2} \int \frac{A}{x-1} - \int \frac{B}{x-2} + \frac{1}{2} \int \frac{C}{x-3} =$$

$$= \frac{1}{2} \ln|x-1| - \ln|x-2| + \frac{1}{2} \ln|x-3| + C = \ln \frac{\sqrt{|x-1| \cdot |x-3|}}{|x-2|} + C$$

**Esercizio 20**

$$\int \frac{2x^2 - 3}{(x-1)(x+1)^2} dx$$



Decomponiamo nel modo che segue

$$\frac{2x^2 - 3}{(x-1)(x+1)^2} = \frac{A}{x-1} + \frac{Bx+C}{(x+1)^2} \quad \text{ottenendo}$$

$$\frac{Ax^2 + 2Ax + A + Bx^2 - Bx + Cx - C}{(x-1)(x+1)^2} = \frac{(A+B)x^2 + (2A-B+C)x + (A-C)}{(x-1)(x+1)^2}$$

$$\begin{cases} A+B=2 \\ 2A-B+C=0 \\ A-C=-3 \end{cases} \quad \begin{cases} C-3+B=2 \\ 2C-6-B+C=0 \\ A=C-3 \end{cases} \quad \begin{cases} C+B=5 \rightarrow C=5-B \\ 3C=B+6 \end{cases}$$

$$3(5-B) = B+6 \rightarrow 15-3B = B+6 \rightarrow 4B=9 \rightarrow B = \frac{9}{4}$$

$$C = 5 - B = 5 - \frac{9}{4} \rightarrow C = \frac{11}{4} \quad A = 2 - B \rightarrow A = 2 - \frac{9}{4} \rightarrow A = -\frac{1}{4}$$

$$\int \frac{2x^2 - 3}{(x-1)(x+1)^2} dx = -\frac{1}{4} \int \frac{dx}{x-1} + \int \frac{\frac{9}{4}x + \frac{11}{4}}{(x+1)^2} dx = -\frac{1}{4} \int \frac{dx}{x-1} + \frac{1}{4} \int \frac{9x+11}{(x+1)^2} dx =$$

$$= -\frac{1}{4} \ln|x-1| + \frac{1}{4} \int \frac{9x+9+2}{(x+1)^2} dx = -\frac{1}{4} \ln|x-1| + \frac{1}{4} \int \frac{9(x+1)+2}{(x+1)^2} dx =$$

$$= -\frac{1}{4} \ln|x-1| + \frac{9}{4} \int \frac{dx}{x+1} + \frac{2}{4} \int \frac{dx}{(x+1)^2} = -\frac{1}{4} \ln|x-1| + \frac{9}{4} \ln|x+1| + \frac{1}{2(x+1)} + C$$